www.mtg.in | February 2021 | Pages 76 | ₹ 40

CONCEPT BOOSTERS Class XI & XII

GEAR UP FOR 2021 JEE MAIN

CBSE warm-up! Class XI & XII

MATHEMATICS today

India's #1
MATHEMATICS MONTHLY
for JEE (Main & Advanced)

MONTHLY TEST DRIVE
CLASS XI & XII

OLYMPIAD CORNER

PRACTICE PAPER 2021
JEE ADVANCED

CONCEPT MAP
CLASS XI & XII

You Ask ? We Answer ✓

mtG
Trust of more than
1 Crore Readers
Since 1982

2021020012976

ALLEN Career Institute

10 TIMES ALL INDIA RANK-1 | ...बात है भरोसे की

in IIT-JEE & Pre-Medical Entrance Exams in Last 11 Years From Classroom

720
720
AIPMT 2010
Lokesh Agarwal
NEET 2013
Ayush Goel
NEET 2019
Nalin Khandelwal
IIT-JEE 2014
Chitraang Murdia
NEET 2020
Soyeb Aftab
NEET 2016
Het Sanjay Shah
IIT-JEE 2019
Kartikey Gupta
IIT-JEE 2016
Aman Bansal
AIPMT 2014
Tejaswin Jha
AIIMS 2017
Nishita Purohit

Join The League of Toppers

Admission Announcement (Online+Offline/Online)

JEE (Main+Adv.) | JEE (Main) | NEET (UG)
NTSE | OLYMPIADS | PNCF | Class 6th to 12th & 12th Pass
GLOBAL STUDIES | EARLY CHILDHOOD DEVELOPMENT (Class HKG to 8th)

For Admission Scholarship Test, Early Fee Benefits, Course & Admission Process, logon to www.allen.ac.in or Call 0744-2757575

ALLEN® CAREER INSTITUTE KOTA (RAJASTHAN) path to success

Corporate Office :
"SANKALP", CP-6, Indra Vihar, Kota (Raj.), India, 324005
Tel.: 0744-2757575 | Web.: www.allen.ac.in | E-mail : info@allen.ac.in

Centers : Kota | Ahmedabad | Bengaluru | Bhilwara | Bhubaneswar | Chandigarh | Chennai | Gandhinagar | Guwahati | Indore | Jaipur Kochi | Mohali | Mumbai | Mysuru| | Nagpur | Panchkula | Pune | Rajkot | Ranchi | Rawatbhata | Sikar | Srinagar | Surat | Ujjain | Vadodara
Overseas : Bahrain | Kuwait | Oman | Qatar | Saudi Arabia | UAE

MATHEMATICS today

Vol. XXXIX No. 2 February 2021

Corporate Office:
Plot 99, Sector 44 Institutional Area,
Gurugram -122 003 (HR), Tel : 0124-6601200
e-mail : info@mtg.in website : www.mtg.in

Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital,
Ring Road, New Delhi - 110029.
Managing Editor : Mahabir Singh
Editor : Anil Ahlawat

CONTENTS

Competition Edge

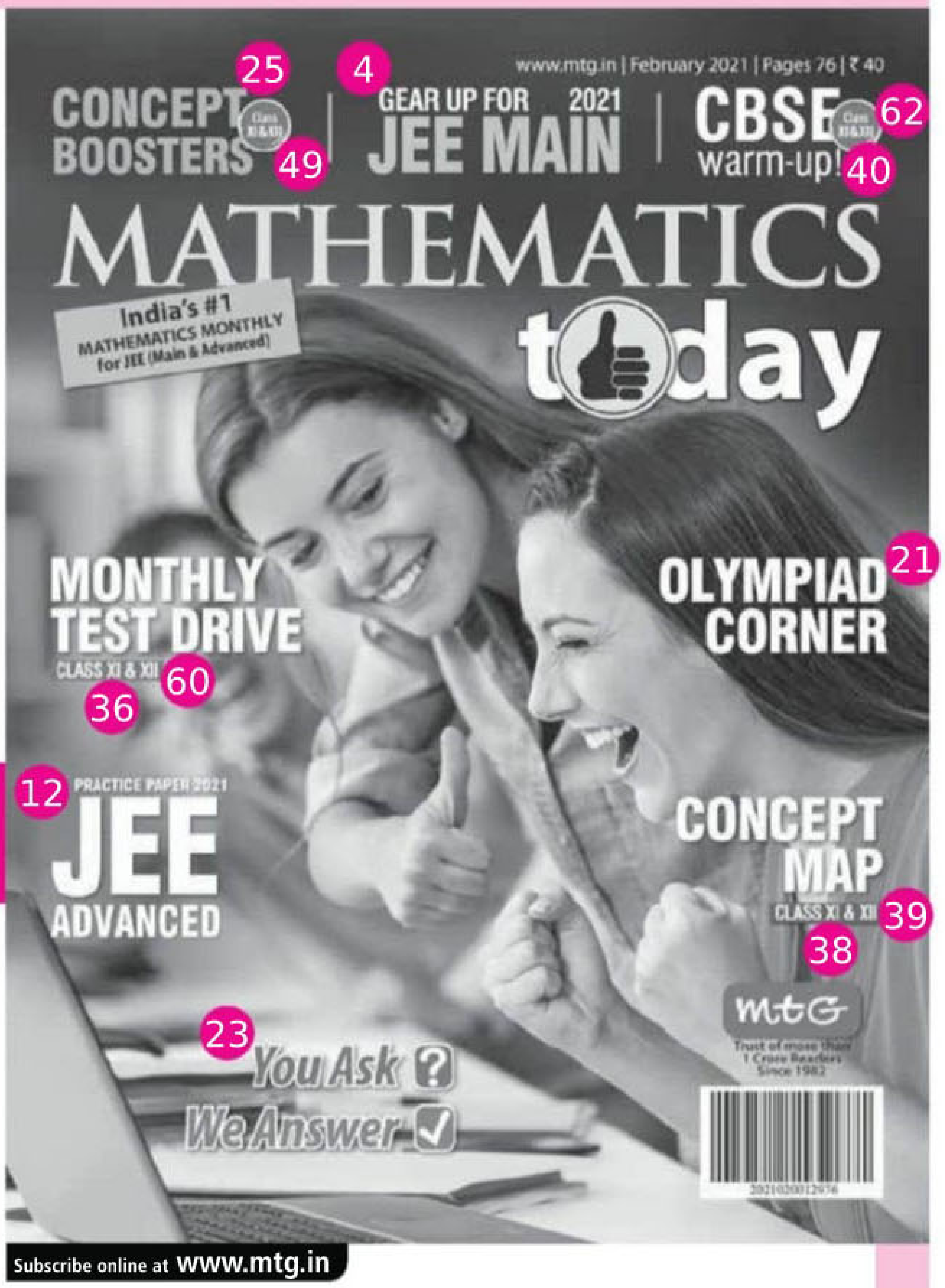
Gear Up for JEE Main
Practice Paper
2 JEE Advanced
Practice Paper
1 Olympiad Corner
3 You Ask We Answer

Class XI

5 Concept Booster
Parabola
6 Monthly Test Drive (Series 9)
Conic Sections
8 Concept Map
Ellipse
0 CBSE warm-up!
Practice Paper

Class XII

9 Concept Map
Differential Equations
9 Concept Booster
Integration
0 Monthly Test Drive (Series 9)
Vector Algebra and
Three Dimensional Geometry
2 CBSE warm-up!
Practice Paper


www.mtg.in | February 2021 | Pages 76 | ₹ 40
25
CONCEPT BOOSTERS
Class XI & XII
49
4
GEAR UP FOR 2021 JEE MAIN
CBSE warm-up!
Class XI & XII
62
40
MATHEMATICS today
India's #1 MATHEMATICS MONTHLY for JEE (Main & Advanced)
MONTHLY TEST DRIVE
CLASS XI & XII
60
36
OLYMPIAD CORNER
21
12
PRACTICE PAPER 2021
JEE ADVANCED
CONCEPT MAP
CLASS XI & XII
39
38
23
You Ask ? We Answer ✓
MTG
Trust of more than 1 Crore Readers Since 1982
2021020012976

Subscribe online at **www.mtg.in**

Individual Subscription Rates

	9 months	15 months	27 months
Mathematics Today	300	500	850
Chemistry Today	300	500	850
Physics For You	300	500	850
Biology Today	300	500	850

Combined Subscription Rates

	9 months	15 months	27 months
PCM	900	1400	2500
PCB	900	1400	2500
PCMB	1200	1900	3400

Send D.D/M.O in favour of MTG Learning Media (P) Ltd.
Payments should be made directly to : MTG Learning Media (P) Ltd,
Plot 99, Sector 44 Institutional Area, Gurugram - 122 003, Haryana.
We have not appointed any subscription agent.

Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.
Editor : Anil Ahlawat

Readers are adviced to make appropriate thorough enquiries before acting upon any advertisements published in this magazine. Focus/Infocus features are marketing incentives. MTG does not vouch or subscribe to the claims and representations made by advertisers. All disputes are subject to Delhi jurisdiction only.
Copyright© MTG Learning Media (P) Ltd.
All rights reserved. Reproduction in any form is prohibited.

GEAR UP for JEE MAIN 2021

Exam Dates 2021
23rd to 26th February; 15th to 18th March
27th to 30th April; 24th to 28th May

Section A will be of Multiple Choice Questions (MCQs). Section B will contain questions whose answers are to be filled in as a Numerical Value. In Section B candidates have to attempt any five questions out of 10.

SECTION A (MULTIPLE CHOICE QUESTIONS)

1. If $\operatorname{Re}\left(\frac{z-8i}{z+6}\right)=0$, then z lies on the curve

(a) $x^2+y^2+6x-8y=0$
(b) $4x-3y+24=0$
(c) $x^2+y^2-8=0$ (d) none of these

2. If the ratio of the roots of $ax^2+2bx+c=0$ is same as the ratio of the $px^2+2qx+r=0$, then

(a) $\frac{2b}{ac}=\frac{q^2}{pr}$ (b) $\frac{b}{ac}=\frac{q}{pr}$
(c) $\frac{b^2}{ac}=\frac{q^2}{pr}$ (d) none of these

3. If the matrices, A, B, $(A+B)$ are non-singular, then $[A(A+B)^{-1}B]^{-1}$ is

(a) $A^{-1}B^{-1}$ (b) $B^{-1}+A^{-1}$
(c) $B^{-1}A^{-1}$ (d) none of these

4. Total number of 6-digit numbers that can be formed, having the property that every succeeding digit is greater than the preceding digit, is equal to

(a) ${}^9C_3$ (b) ${}^{10}C_3$ (c) ${}^9P_3$ (d) ${}^{10}P_3$

5. Range of $f(x)=\sin^{-1}\left(\sqrt{x^2+x+1}\right)$ is

(a) $\left(0,\frac{\pi}{2}\right]$ (b) $\left(0,\frac{\pi}{3}\right]$
(c) $\left[\frac{\pi}{3},\frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$

6. Let $g(x)$ be a polynomial of degree one and $f(x)$ be defined by $f(x)=\begin{cases} g(x), & x\le 0 \\ |x|^{\sin x}, & x>0 \end{cases}$. If $f(x)$ is continuous satisfying $f'(1)=f(-1)$, then $g(1)$ is

(a) $(1+\sin 1)x+1$ (b) $(1-\sin 1)x+1$
(c) $(1-\sin 1)x-1$ (d) $(1+\sin 1)x-1$

7. If $3(a+2c)=4(b+3d)$, then the equation $ax^3+bx^2+cx+d=0$ will have

(a) no real solution
(b) at least one real root in $(-1, 0)$
(c) at least one real root in $(0, 1)$
(d) none of these

8. If $\int x^5(1+x^3)^{2/3}dx=A(1+x^3)^{8/3}+B(1+x^3)^{5/3}+c$, then

(a) $A=\frac{1}{4}, B=\frac{1}{5}$ (b) $A=\frac{1}{8}, B=-\frac{1}{5}$
(c) $A=-\frac{1}{8}, B=\frac{1}{5}$ (d) none of these

9. The points (a, b), (c, d) and $\left(\frac{kc+la}{k+1},\frac{kd+lb}{k+1}\right)$ are

(a) vertices of an equilateral triangle
(b) vertices of an isosceles triangle
(c) vertices of a right angled triangle
(d) collinear

10. The circles having radii r_1 and r_2 intersect orthogonally. Length of their common chord is

(a) $\frac{\sqrt{r_1^2+r_2^2}}{2r_1r_2}$ (b) $\frac{\sqrt{r_1^2+r_2^2}}{r_1r_2}$
(c) $\frac{r_1r_2}{\sqrt{r_1^2+r_2^2}}$ (d) $\frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}}$

11. A straight line $\vec{r}=\vec{a}+\lambda\vec{b}$ meets the plane $\vec{r}\cdot\vec{n}=0$ in P. The position vector of P is

(a) $\vec{a}+\frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{b}$ (b) $\vec{b}-\frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{a}$
(c) $\vec{a}-\frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{b}$ (d) none of these

12. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors, then

(a) $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{b} & \vec{c} & \vec{a} \\ \vec{c} & \vec{a} & \vec{b} \end{vmatrix} = \vec{0}$ (b) $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \end{vmatrix} = \vec{0}$

(c) $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{c} & \vec{b}\cdot\vec{b} \end{vmatrix} = \vec{0}$

(d) $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{c} & \vec{c}\cdot\vec{b} \end{vmatrix} = \vec{0}$

13. If the letters of the word 'REGULATIONS' be arranged at random, then the probability that there will be exactly 4 letters between the R and the E is

(a) $\frac{{}^9C_4\, 6!}{11!}$ (b) $\frac{{}^9C_5\, 6!2!}{12!}$

(c) $\frac{{}^9P_4\, 6!2!}{11!}$ (d) $\frac{{}^9C_4\, 6!2!4!}{11!}$

14. The points of intersection of the two ellipses
$x^2 + 2y^2 - 6x - 12y + 23 = 0$ and
$4x^2 + 2y^2 - 20x - 12y + 35 = 0$.

(a) lie on a circle centred at (8/3, 3) and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$

(b) lie on a circle centred at (–8/3, 3) and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$

(c) lie on a circle centred at (8, 9) and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$

(d) are not cyclic

15. Equation of a plane which passes through the point of intersection of lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and at greatest distance from the origin is

(a) $7x + 2y + 4z = 54$ (b) $4x + 3y + 5z = 50$

(c) $3x + 4y + 5z = 49$ (d) $x + y + z = 12$

16. If $f(x) = \begin{cases} \sin^{-1}(\sin x), & x > 0 \\ \frac{\pi}{2}, & x = 0 \\ \cos^{-1}(\cos x), & x < 0 \end{cases}$, then

(a) $x = 0$ is a point of maxima

(b) $x = 0$ is a point of minima

(c) $x = 0$ is point of intersection

(d) none of these

17. If the angle of intersection of the circles $x^2 + y^2 + x + y = 0$ and $x^2 + y^2 + x - y = 0$ is θ, then equation of the line passing through (1, 2) and making an angle θ with the y-axis is

(a) $x = 1$ (b) $y = 2$

(c) $x + y = 3$ (d) $x - y = 3$

18. Let f be a continuous and differentiable function in (a, b), $\lim_{x \to a^+} f(x) \to \infty$ and $\lim_{x \to b^-} f(x) \to -\infty$. If $f'(x) + f^2(x) \geq -1$ for $a < x < b$, then

(a) $b - a \leq \pi$ (b) $b - a \geq \pi$

(c) $b - a = \pi$ (d) none of these

19. If $\frac{\tan 3A}{\tan A} = a$, then $\frac{\sin 3A}{\sin A}$ is equal to

(a) $\frac{2a}{a+1}$ (b) $\frac{2a}{a-1}$ (c) $\frac{a}{a+1}$ (d) $\frac{a}{a-1}$

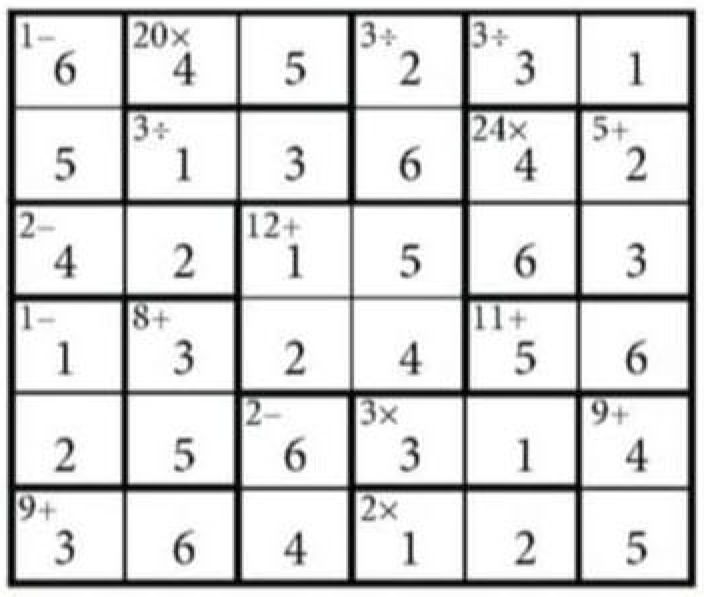
20. The value of $\int_e^{\pi^2} [\log_\pi x]\, d(\log_e x)$ (where [·] denotes greatest integer function) is

(a) $2 \log_e \pi$ (b) $\log_e \pi$

(c) 1 (d) 0

PUZZLE CORNER

ANSWER - JANUARY 2021



1– 6	20× 4	5	3÷ 2	3÷ 3	1
5	3÷ 1	3	6	24× 4	5+ 2
2– 4	2	12+ 1	5	6	3
1– 1	8+ 3	2	4	11+ 5	6
2	5	2– 6	3× 3	1	9+ 4
9+ 3	6	4	2× 1	2	5

SECTION B (NUMERICAL VALUE TYPE)

21. If A is 3 order square matrix such that $|A| = 2$, then $|\text{adj}\,(\text{adj}\,(\text{adj}\,A))|$ is _________.

22. Given $a=\dfrac{x}{y-z}$, $b=\dfrac{y}{z-x}$ and $c=\dfrac{z}{x-y}$, where x, y and z are not all zero, then the value of $1 + ab + bc + ca$ is equal to _______.

23. The integer which divides the number $101^{100} - 1$, is _______.

24. $\lim\limits_{x\to 1}\left[\text{cosec}\,\dfrac{\pi x}{2}\right]^{1/1-x}$ is equal to (where $[\cdot]$ represents the greatest integer function) _________.

25. If $f(0) = 0$, $f'(0) = 2$, then the derivative of $y = f(f(f(f(x))))$ at $x = 0$ is _________.

26. If $\int_0^1 \cot^{-1}(1-x+x^2)dx = \lambda\int_0^1 \tan^{-1} x dx$, then '$\lambda$' is equal to _________.

27. Normals AO, AA_1, AA_2 are drawn to parabola $y^2 = 8x$ from the point $A(h, 0)$. If triangle OA_1A_2 is equilateral, then possible values of h is _________.

28. Let $z = x + iy$ be a complex numbers where x and y are integers. Then area of the rectangle whose vertices are roots of $z\bar{z}^3 + \bar{z}z^3 = 738$ is _________.

29. In a hurdle race, a runner has probability p of jumping over a specific hurdle. Given that in 5 trials, the runner succeeded 3 times, the conditional probability that the runner had succeeded in the first trial is _________.

30. If $f(n)=\sum\limits_{k=1}^{n}\sum\limits_{j=k}^{n}({}^nC_j)({}^jC_k)$, then the value of $f(6)$ is _________.

SOLUTIONS

1. (a) : Let $z = x + iy$. Then

$$\frac{z-8\text{i}}{z+6}=\frac{x+(y-8)\text{i}}{(x+6)+\text{i}y}=\frac{(x+(y-8)\text{i})(x+6-\text{i}y)}{(x+6)^2+y^2}$$

$$=\frac{(x^2+6x+y^2-8y)+\text{i}(xy-8x+6y-48-xy)}{(x+6)^2+y^2}$$

$$\Rightarrow\ \text{Re}\left(\frac{z-8\text{i}}{z+6}\right)=0\ \Rightarrow\ x^2+y^2+6x-8y=0$$

2. (c) : Let roots of the equation $ax^2 + 2bx + c = 0$ are α and β and roots of the equation $px^2 + 2qx + r = 0$ are γ and δ.

Given, $\dfrac{\alpha}{\beta}=\dfrac{\gamma}{\delta}\ \Rightarrow\ \dfrac{\alpha}{\gamma}=\dfrac{\beta}{\delta}$

$$\Rightarrow\ \frac{\alpha+\beta}{\gamma+\delta}=\sqrt{\frac{\alpha\beta}{\gamma\delta}}\ \Rightarrow\ \frac{-\frac{2b}{a}}{-\frac{2q}{p}}=\sqrt{\frac{\frac{c}{a}}{\frac{r}{p}}}\ \Rightarrow\ \frac{b^2}{ac}=\frac{q^2}{pr}$$

3. (b) : $[A(A+B)^{-1}B]^{-1} = B^{-1}((A+B)^{-1})^{-1}A^{-1}$

$= B^{-1}(A+B)A^{-1}$ $\qquad [\because (A^{-1})^{-1} = A]$

$= (B^{-1}A + I)A^{-1} = B^{-1}I + IA^{-1} = B^{-1} + A^{-1}$

4. (a) : $x_1 < x_2 < x_3 < x_4 < x_5 < x_6$, when the number is $x_1x_2x_3x_4x_5x_6$

Clearly, no digit can be zero.

Also, all the digits are distinct. So, let us first select six digits from the list of digits 1, 2, 3, 4, 5, 6, 7, 8, 9 which can be done in ${}^9C_6$ ways.

After selecting these digits, they can be put only in one order.

Thus, total number of such numbers

$= {}^9C_6 \times 1 = {}^9C_6 = {}^9C_3$

5. (c) : For domain point of view, $0 \le x^2 + x + 1 \le 1$,

but $x^2+x+1\ge\dfrac{3}{4}\ \Rightarrow\ \dfrac{\sqrt{3}}{2}\le\sqrt{x^2+x+1}\le 1$

$$\Rightarrow\ \frac{\pi}{3}\le\sin^{-1}(\sqrt{x^2+x+1})\le\frac{\pi}{2}$$

6. (b) : $f(0^+)=\lim\limits_{x\to 0^+}|x|^{\sin x}=1$

$f(0^-) = g(0) = 1$

Let $g(x) = ax + b$

$\Rightarrow\ b = 1\ \Rightarrow\ g(x) = ax + 1$

For $x > 0$, $f'(x)=e^{\sin x\ln(|x|)}\left[\cos x\ln(|x|)+\dfrac{\sin x}{x}\right]$

$f'(1) = 1[0 + \sin 1] = \sin 1$

$f(-1) = -a + 1\ \Rightarrow\ a = 1 - \sin 1$

$g(1) = (1 - \sin 1)x + 1$

7. (b) : Let $f(x)=\dfrac{ax^4}{4}+\dfrac{bx^3}{3}+\dfrac{cx^2}{2}+dx$, which is continuous and differentiable.

SOLUTION SENDERS

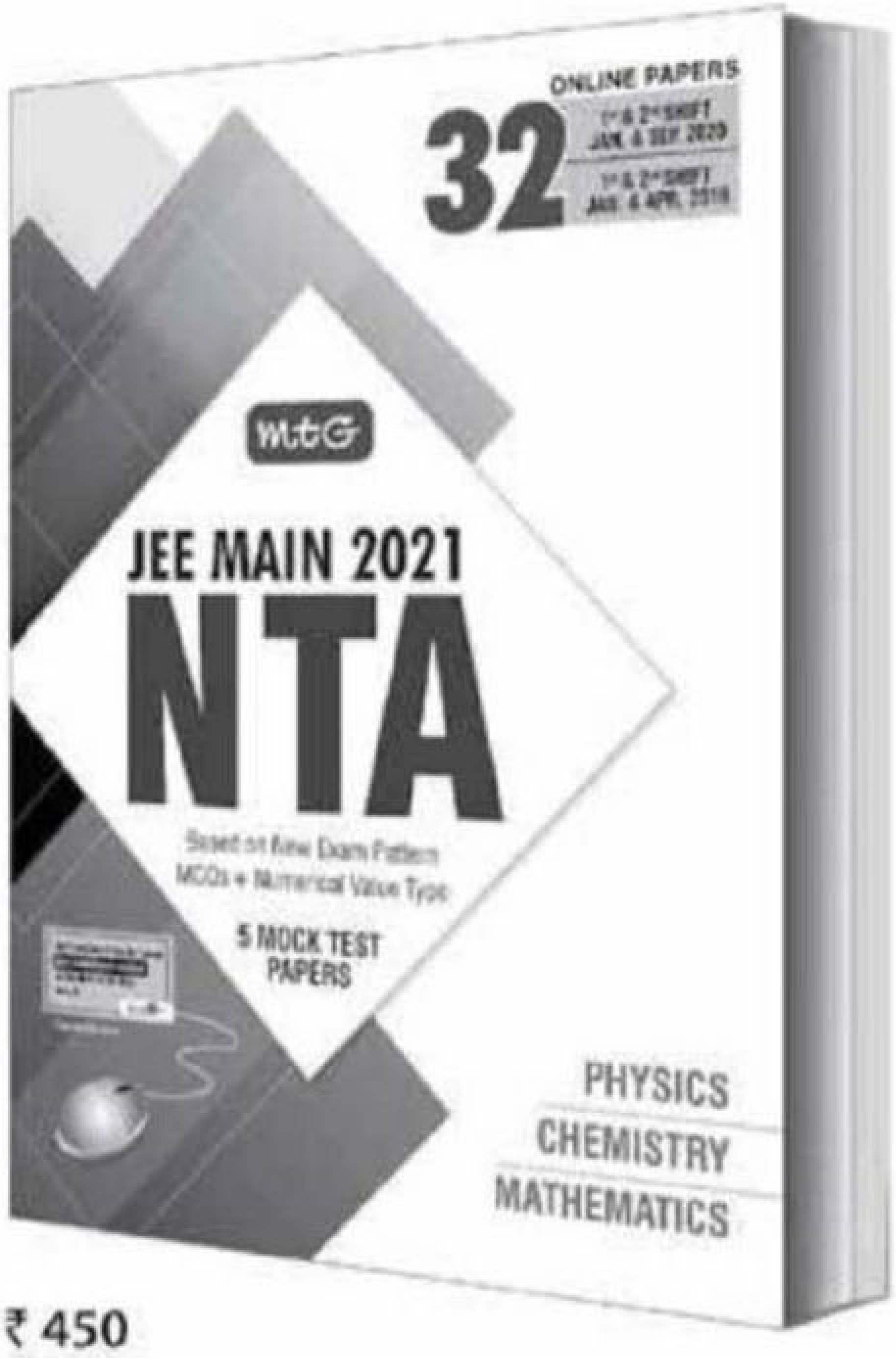
Samurai Sudoku (January)

- Nitin Singla (Gujarat)

Mathdoku (January)

- Anirudh Srivatsa

Reach the peak of readiness for JEE Main 2021



₹ 450

WITH SEPTEMBER 2020 ONLINE PAPERS

The Wait is Over! MTG Presents you all the 16 Fully Solved Question Papers of **JEE Main (I) January and JEE Main (II) September 2020** & 16 Fully Solved Question Papers of **JEE Main (I) January** and **JEE Main (II) April, 2019** conducted by NTA with Chapterwise graphical analysis showing the weightage of chapters and Mock Test Papers as per latest pattern of JEE Main. Get the maximum benefit from the book through personalised course on **Pedagogy app**. The best preparation for tomorrow is doing your best today so, go and get your copy.

Highlights

- Fully Solved Authentic Papers
- 16 (Jan. & Sep., 2020) + 16 (Jan. & Apr., 2019) Online Papers
- 2,575 MCQs for Practice
- Chapterwise Tabular and Graphical Analysis Showing the Weightage of Chapters
- 5 MTPs as per the latest JEE Main pattern

Practice Online Tests to Predict Your Rank

Make It Happen, CRACK JEE Main

Get the complete feel of Online exam by attempting tests on computers

Take the Final Leap Towards ***JEE Main*** Success!

Available at all leading book shops throughout the country. To buy online visit www.mtg.in.
For more information or for help in placing your order, call 0124-6601200 or email: *info@mtg.in*

$$f(0) = 0,\ f(-1) = \frac{a}{4} - \frac{b}{3} + \frac{c}{2} - d$$

$$= \frac{1}{4}(a+2c) - \frac{1}{3}(b+3d) = 0$$

So, according to Rolle's theorem, there exist at least one root of $f'(x) = 0$ in $(-1, 0)$.

8. (b) : Let $I = \int x^5(1+x^3)^{2/3}dx$

Let $1 + x^3 = t^2$ and $3x^2\,dx = 2t\,dt$

$$\therefore \int x^5(1+x^3)^{2/3}dx = \int x^3(1+x^3)^{2/3}x^2\,dx$$

$$= \int (t^2-1)(t^2)^{2/3}\frac{2}{3}t\,dt = \frac{2}{3}\int (t^2-1)t^{7/3}dt$$

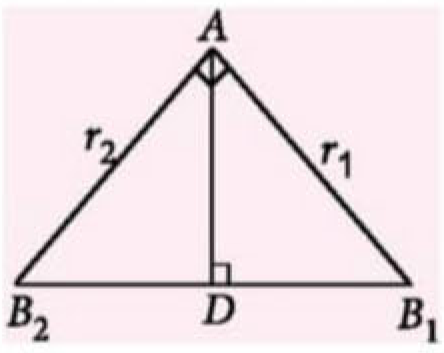
$$= \frac{2}{3}\int (t^{13/3} - t^{7/3})\,dt = \frac{2}{3}\left\{\frac{3}{16}t^{16/3} - \frac{3}{10}t^{10/3}\right\} + c$$

$$= \frac{1}{8}(1+x^3)^{8/3} - \frac{1}{5}(1+x^3)^{5/3} + c$$

9. (d) : The given points are collinear because the point $\left(\frac{kc+\mathrm{l}a}{k+1}, \frac{kd+\mathrm{l}b}{k+1}\right)$ divides the points (a, b) and (c, d) in the ratio of $k : l$.

10. (d) : Let $\angle AB_1B_2 = \theta$

$\Rightarrow \quad AD = r_1\sin\theta$ and $AD = r_2\cos\theta$



$$\Rightarrow \quad AD^2\left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right) = 1$$

$$\Rightarrow \quad AD = \frac{r_1r_2}{\sqrt{r_1^2 + r_2^2}}$$

Thus, length of common chord $= \dfrac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}}$

11. (c) : The straight line $\vec{r} = \vec{a} + \lambda\vec{b}$ meets the plane $\vec{r}\cdot\vec{n} = 0$ in P for which λ is given by

$$(\vec{a} + \lambda\vec{b})\cdot\vec{n} = 0 \quad\Rightarrow\quad \lambda = -\frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}$$

Thus, the position vector of P is $\vec{r} = \vec{a} - \dfrac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{b}$

[Putting the value of λ in $\vec{r} = \vec{a} + \lambda\vec{b}$]

12. (b) : Since $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, there exists $(x, y, z$ not all zero) such that

$$x\vec{a} + y\vec{b} + z\vec{c} = 0 \qquad \text{... (i)}$$

Multiply by $\vec{a}$ and $\vec{b}$ scalarly, we get

$$x(\vec{a}\cdot\vec{a}) + y(\vec{a}\cdot\vec{b}) + z(\vec{a}\cdot\vec{c}) = 0 \qquad \text{... (ii)}$$

and $x(\vec{b}\cdot\vec{a}) + y(\vec{b}\cdot\vec{b}) + z(\vec{b}\cdot\vec{c}) = 0$

Eliminating x, y and z from (i), (ii) and (iii), we get

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \end{vmatrix} = 0$$

13. (d) : Total number of ways of arranging 11 letters $= (11)!$

Selection of 4 letters to be placed between R and E from remaining $9 = {}^9C_4$

These 4 letters can be permuted in 4! ways.

Now permutation of $(\text{R} \underbrace{\bullet\bullet\bullet\bullet}_{\text{4 letters}} \text{E}) \underbrace{\bullet\bullet\bullet\bullet\bullet}_{\text{5 letters}} = 6!$

R and E can be interchanged in 2! ways

$\therefore \quad$ Number of favourable ways $= {}^9C_4 \cdot 6! \cdot 2!\ 4!$

Hence, the required probability is $\dfrac{{}^9C_4\, 6!2!4!}{11!}$.

14. (a) : Equation of any curve passing through the intersection of the given ellipses is given by

$$4x^2 + 2y^2 - 20x - 12y + 35 + \lambda(x^2 + 2y^2 - 6x - 12y + 23) = 0$$

which represents a circle if

$$4 + \lambda = 2 + 2\lambda \quad\Rightarrow\quad \lambda = 2$$

Thus, the equation of the circle is,

$$6x^2 + 6y^2 - 32x - 36y + 81 = 0$$

$$\Rightarrow \quad x^2 + y^2 - \left(\frac{16}{3}\right)x - 6y + \frac{81}{6} = 0$$

Centre of the circle is (8/3, 3)

and the radius is $\sqrt{\left(\dfrac{8}{3}\right)^2 + (3)^2 - \dfrac{81}{6}}$

$$= \sqrt{\frac{128+162-243}{18}} = \frac{1}{3}\sqrt{\frac{47}{2}}$$

15. (b) : Any point on the first line is $P(3\lambda + 1, \lambda + 2, 2\lambda + 3)$ and on the second line is $Q(\mu + 3, 2\mu + 1, 3\mu + 2)$. P and Q represent the same point if $\lambda = \mu = 1$ and the point of intersection of the given lines is $P(4, 3, 5)$. The planes given in (a), (b), (c) and (d) all pass through P, the plane at greatest distance is one which is at a distance equal to OP from the origin. So the distance of the plane from the origin is

$$\sqrt{4^2 + 3^2 + 5^2} = \sqrt{50}$$

which is satisfied by the plane is $4x + 3y + 5z = 50$.

16. (a) : $f(0) = \pi/2$, $f(0^+) = 0$, $f(0^-) = 0$.

Hence, $x = 0$ is the point of maxima.

17. (b) : Let A and B be the centres and r_1 and r_2 the radii of the two circles, then

$$A \equiv \left(-\frac{1}{2}, -\frac{1}{2}\right),\ B = \left(-\frac{1}{2}, \frac{1}{2}\right),\ r_1 = \frac{1}{\sqrt{2}},\ r_2 = \frac{1}{\sqrt{2}}$$

$$\cos\theta = \frac{r_1^2 + r_2^2 - AB^2}{2r_1r_2} = \frac{\frac{1}{2} + \frac{1}{2} - 1}{2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = 0$$

$\therefore\quad \theta = \pi/2$

Required line is parallel to the x-axis and since it passes through (1, 2), its equation will be $y = 2$.

18. (b) : We have, $f'(x) + f^2(x) \ge -1$

$$\Rightarrow \frac{f'(x)}{1 + f^2(x)} + 1 \ge 0 \text{ for } x \in (a, b) \qquad \ldots \text{(i)}$$

$$\frac{d}{dx}(\tan^{-1} f(x) + x) = \frac{f'(x)}{1 + (f(x))^2} + 1 \ge 0 \qquad \text{[From (i)]}$$

$\Rightarrow$ $h(x) = \tan^{-1} f(x) + x$, is a non-decreasing function in the interval (a, b).

$$\Rightarrow \lim_{x\to a} h(x) \le \lim_{x\to b} h(x)$$

$$\Rightarrow \lim_{x\to a^+} (\tan^{-1} f(x) + x) \le \lim_{x\to b^-} (\tan^{-1} f(x) + x)$$

$\Rightarrow \frac{\pi}{2} + a \le -\frac{\pi}{2} + b$. Hence, $b - a \ge \pi$

19. (b) : Given, $\frac{\tan 3A}{\tan A} = a \Rightarrow \frac{3\tan A - \tan^3 A}{\tan A(1 - 3\tan^2 A)} = a$

$\Rightarrow\quad 3 - \tan^2 A = a - 3a \tan^2 A$

$\Rightarrow\quad \tan^2 A(3a - 1) = a - 3$

$$\Rightarrow \tan^2 A = \frac{a-3}{3a-1} \therefore \sin^2 A = \frac{a-3}{4(a-1)}$$

$$\text{Now, } \frac{\sin 3A}{\sin A} = \frac{3\sin A - 4\sin^3 A}{\sin A}$$

$$= 3 - 4\sin^2 A = 3 - 4\left(\frac{a-3}{4(a-1)}\right) = \frac{3a - 3 - a + 3}{(a-1)} = \frac{2a}{(a-1)}$$

20. (b) : Let $\log_e x = t$

$$\therefore \quad I = \int_1^{\log_e \pi^2} [\log_\pi e^t]dt = \int_1^{\log_e \pi^2} [t \log_\pi e]dt$$

$$= \int_1^{\log_e \pi} 0\,dt + \int_{\log_e \pi}^{2\log_e \pi} 1\,dt = \log_e \pi$$

21. (256) : We know that $|\text{adj}(\text{adj } A)| = |A|^{(n-1)2}$

$\Rightarrow\quad |\text{adj}(\text{adj } (\text{adj } A))| = |\text{adj A}|^{(n-1)2} = |A|^{(n-1)3}$

$= 2^8 = 256$

22. (0) : $a = x/(y - z) \Rightarrow x - ay + az = 0$

$b = y/(z - x) \Rightarrow bx + y - bz = 0$

$c = z/(x - y) \Rightarrow -cx + cy + z = 0$

As x, y, z are not all zero, thus the above system has a non-trivial solution

$$\therefore \quad \Delta = \begin{vmatrix} 1 & -a & a \\ b & 1 & -b \\ -c & c & 1 \end{vmatrix} = 0 \Rightarrow 1 + ab + bc + ca = 0$$

23. (10000) : $(1 + 100)^{100} = 1 + 100 \cdot 100$

$$+ \frac{100 \cdot 99}{1 \cdot 2} \cdot (100)^2 + \frac{100 \cdot 99 \cdot 98}{1 \cdot 2 \cdot 3}(100)^3 + \ldots\ldots$$

$\Rightarrow\quad (101)^{100} - 1$

$$= 100 \cdot 100\left[1 + \frac{100 \cdot 99}{1 \cdot 2} + \frac{100 \cdot 99 \cdot 98}{1 \cdot 2 \cdot 3} \cdot 100 + \ldots\ldots\right]$$

From above it is clear that

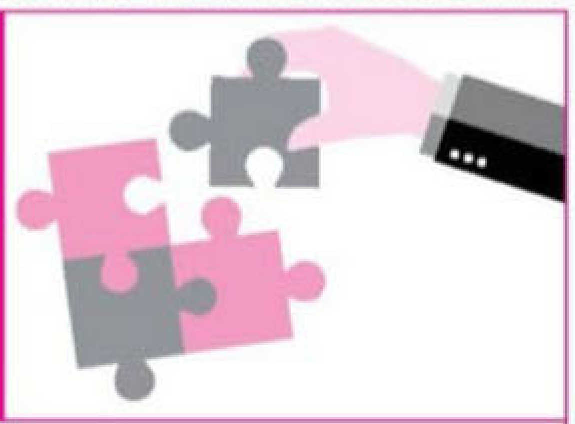
$(101)^{100} - 1$ is divisible by $(100)^2 = 10000$

24. (1) : $\operatorname{cosec} \frac{\pi x}{2} \to 1$ when $x \to 1$

$\therefore\quad$ Limit = 1

PUZZLE

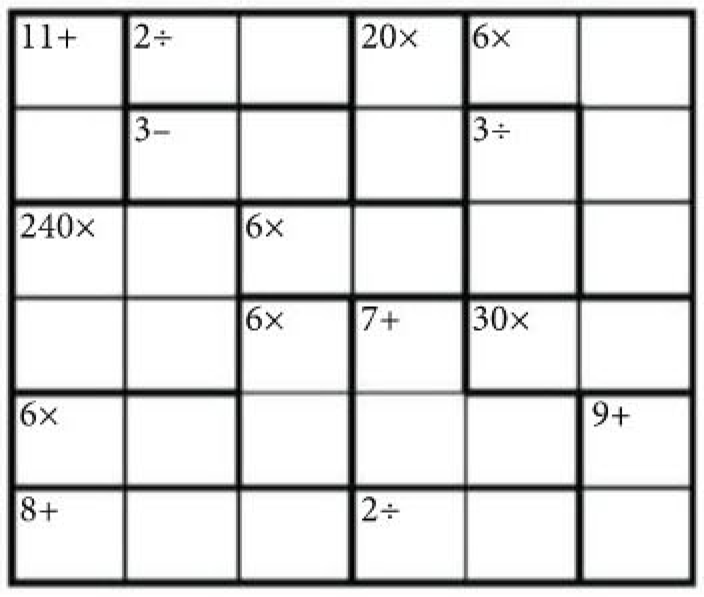
CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



11+	2÷		20×	6×	
	3–			3÷	
240×		6×			
		6×	7+	30×	
6×					9+
8+			2÷		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

25. (16) : $y'(x) = f'(f(f(f(x))))\,f'(f(f(x)))\,f'(f(x))f'(x)$

$\Rightarrow\ y'(0) = f'(f(f(f(0))))\,f'(f(f(0)))\,f'(f(0))\,f'(0)$

$= f'(f(f(0)))\,f'(f(0))\,f'(0)\,f'(0)$

$= f'(f(0))\,f'(0)\,f'(0)\,f'(0)$

$= f'(0)\,f'(0)\,f'(0)\,f'(0) = (f'(0))^4 = 2^4 = 16$

26. (2) : $\int_0^1 \cot^{-1}(1-x+x^2)\,dx = \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$

$= \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx$

$= \int_0^1 \tan^{-1} x\,dx + \int_0^1 \tan^{-1}(1-x)\,dx$

$= \int_0^1 \tan^{-1} x\,dx + \int_0^1 \tan^{-1}[1-(1-x)]\,dx = 2\int_0^1 \tan^{-1} x\,dx$

$\Rightarrow\ \lambda = 2$

27. (28) : Let $A_1 \equiv (2t_1^2, 4t_1)$, $A_2 \equiv (2t_1^2, -4t_1)$

Clearly, $\angle A_1OA = \pi/6$

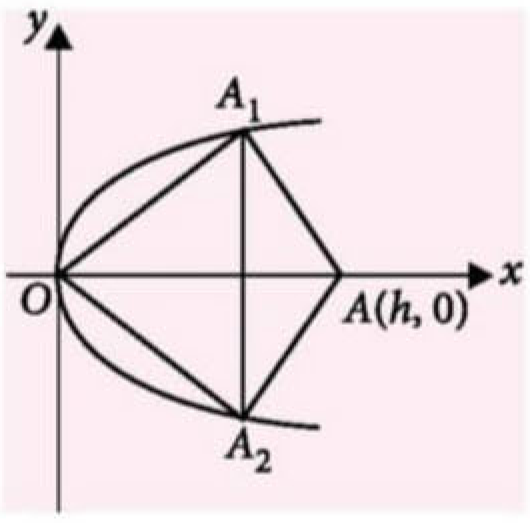
$\Rightarrow\ \frac{2}{t_1} = \frac{1}{\sqrt{3}} \quad \Rightarrow\ t_1 = 2\sqrt{3}$

Equation of normal at A_1 is

$y = -t_1x + 4t_1 + 2t_1^3$

$\Rightarrow\ h = 4 + 2t_1^2$

$= 4 + 2\cdot 12 = 28$



28. (80) : We have, $z\bar{z}^3 + \bar{z}z^3 = 738$

$\Rightarrow\ z\bar{z}(\bar{z}^2 + z^2) = 738$

$\Rightarrow\ (x^2 + y^2)[(x - iy)^2 + (x + iy)^2] = 738$

$\Rightarrow\ (x^2 + y^2)(2x^2 - 2y^2) = 738$

$\Rightarrow\ x^4 - y^4 = 369$

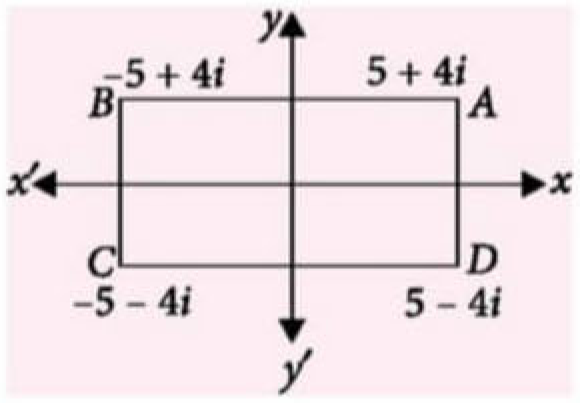
$\Rightarrow\ x^4 \geq 369$

$\Rightarrow\ |x| \geq 4$

Let us try $x = \pm 5$

$\Rightarrow\ 625 - 369 = y^4$

$\Rightarrow\ y = \pm 4$



Thus, roots of $z\bar{z}^3 + \bar{z}z^3 = 738$ are $\pm 5 \pm 4i$.

Area of the rectangle $ABCD$ whose vertices are $\pm 5 \pm 4i$ is (10) (8) = 80

29. (0.6) : Let A denote the event that the runner succeeds exactly 3 times out of five and B denote the event that the runner succeeds on the first trial.

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

But $P(B \cap A) = P$ (succeeding in the first trial and exactly twice in other four trials)

$= p({}^4C_2\, p^2(1 - p)^2) = 6p^3(1 - p)^2$

And $P(A) = {}^5C_3\, p^3\,(1 - p)^2 = 10p^3(1 - p)^2$

Thus, $P(B|A) = \frac{6p^3(1-p)^2}{10p^3(1-p)^2} = \frac{3}{5} = 0.6$

30. (665) : $f(n) = \sum_{j=1}^{n} ({}^nC_j)({}^jC_1) + \sum_{j=2}^{n} ({}^nC_j)({}^jC_2) + \ldots.$

$+ \sum_{j=n-1}^{n} ({}^nC_j)({}^jC_{n-1}) + ({}^nC_n)({}^nC_n)$

$= ({}^nC_1)({}^1C_1) + ({}^nC_2)({}^2C_1 + {}^2C_2) + ({}^nC_3)({}^3C_1 + {}^3C_2 + {}^3C_3)$

$+ \ldots + ({}^nC_n)({}^nC_1 + {}^nC_2 + \ldots + {}^nC_n)$

$= ({}^nC_1)(2 - 1) + ({}^nC_2)(2^2 - 1) + ({}^nC_3)(2^3 - 1) + \ldots$

$+ {}^nC_n\,(2^n - 1)$

$= \{(2 + 1)^n - 1\} - \{(1 + 1)^n - 1\} = 3^n - 2^n$

$\therefore\ f(6) = 665$

◆ ◆

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

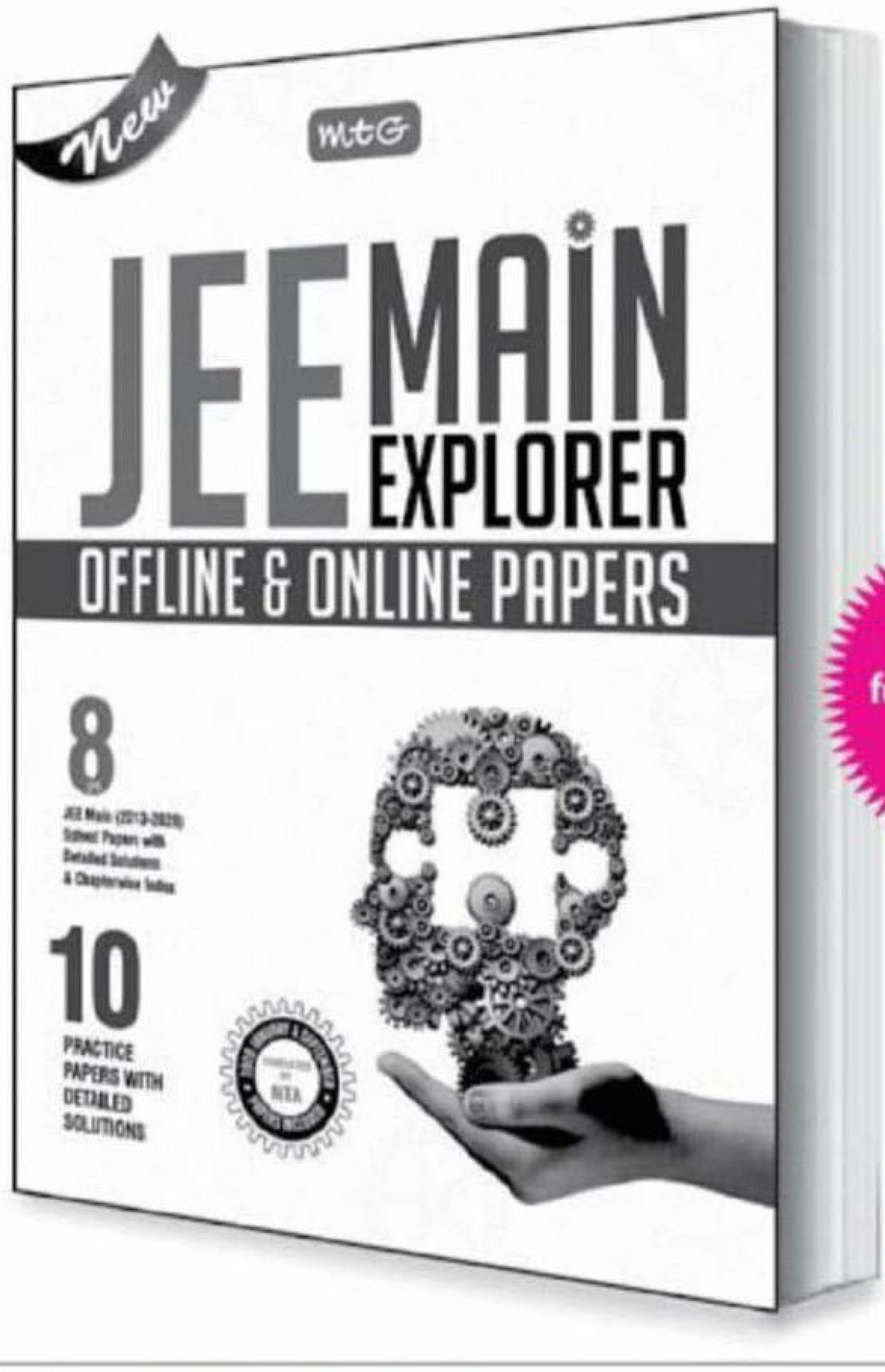
The puzzle has a unique answer.

2	9		7			4						2	8		9			6		
		7	1	9			2						7	3	2		6			9
3				4	6			7								8	7	3	5	
	7				2		9	8				8					9	7		6
9	8				7	2						7				6			2	5
4	3	2	9									1	6		5			8		
8		9		7		3		2		6			9	7	6			2		
		4	3	2			7		3					2		9			7	
7			8					9			7				7			9		1
						9	3						4	6						
						1		4	7					9						
								6		4	9			3						
	1	6	8				9		6	1		8		4		1		2		
	9			5			6	7			5		3	1			8	7		
3		5		2		4			9				6				9		8	1
		3	2	9				5								6	2	5		8
5	6					9	1							2	8			9	6	
8		9				3						6			5				2	7
	5				2			9				1	2				6			
			9	6	3							7				5	1			
9					5	6							5		2			6		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

mtG

What is JEE (MAIN) like?


New
mtG
JEE MAIN EXPLORER
OFFLINE & ONLINE PAPERS
8
JEE Main (2013-2020) Solved Papers with Detailed Solutions & Chapterwise Index
10
PRACTICE PAPERS WITH DETAILED SOLUTIONS
NTA

Visit www.mtg.in for latest offers and to buy online!

₹ 700/-

Thinking about competing in the JEE? Unsure of what it's like? Get a first-hand feel with MTG's JEE Main Explorer. With solved papers from the past 8 years, glance through the changing pattern of examination. Or attempt the papers to check your chances of success. Either way, at ₹ 700, isn't this a steal for JEE aspirants like you? So what are you waiting for? **Order MTG's JEE Main Explorer today.**

Scan now with your smartphone or tablet
Application to read QR codes required

Available at all leading book shops throughout the country. To buy online visit www.mtg.in.

For more information or for help in placing your order, call 0124-6601200 or email: *info@mtg.in*

JEE ADVANCED PRACTICE PAPER

2021
PROBLEM
PROBLEM
PROBLEM
PROBLEM

Exam on 3rd July

PAPER-1

SECTION-1

This section contains SIX (06) questions. Each question has FOUR options. ONLY ONE of these four options is the correct answer.

1. $ABCD$ is a square of side 1 unit. A circle passes through vertices A, B of the square and the remaining two vertices of the square lie outside the circle. The length of the tangent drawn to the circle from vertex D is 2 units. The radius of the circle is

(a) $\sqrt{5}$ (b) $\frac{1}{2}\sqrt{10}$ (c) $\frac{1}{3}\sqrt{12}$ (d) $\sqrt{8}$

2. In the xy-plane, how many straight lines whose x-intercept is a prime number and whose y-intercept is a positive integer, pass through the point (4, 3)?

(a) 1 (b) 2 (c) 3 (d) 4

3. There are four seats numbered 1, 2, 3, 4 in a room and four persons having tickets corresponding to these seats (one person having one ticket). Now the person having the ticket number 1, enters into the room and sits on any of the seat at random. Then the person having the ticket number 2, enters in room. If his seat is empty then he sits on his seat otherwise he sits on any of the empty seat at random. Similarly the other persons sit. Probability that the person having ticket numbered 4 gets the seat number 4 is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

4. Consider $\int_0^x (t^2-8t+13)\,dt = x\sin\left(\frac{a}{x}\right)$ and ($a, x \in R - \{0\}$). x takes the values for which the equation has a solution, then the number of values of $a \in [0, 100]$ is

(a) 1 (b) 2 (c) 3 (d) 4

5. If $\int \frac{\sec^2 x - 2010}{\sin^{2010} x}dx = \frac{P(x)}{\sin^{2010} x} + C$, then value of $P\left(\frac{\pi}{3}\right)$ is

(a) 0 (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\sqrt{2}$

6. Sum of the series
${}^nC_1 \sin a + {}^nC_2 \sin 2a + ... + {}^nC_n \sin na$ is equal to

(a) $2^n \cos^n\left(\frac{a}{2}\right)\cdot\sin\left(\frac{na}{2}\right)$ (b) $2^n \sin^n\left(\frac{a}{2}\right)\cdot\cos\left(\frac{na}{2}\right)$

(c) $2^n \sin^n\left(\frac{a}{2}\right)\cdot\sin\left(\frac{na}{2}\right)$ (d) $2^n \cos^n\left(\frac{a}{2}\right)\cdot\cos\left(\frac{na}{2}\right)$

SECTION-2

This section contains SIX (06) questions. Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).

7. The equation $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ represents a hyperbola with

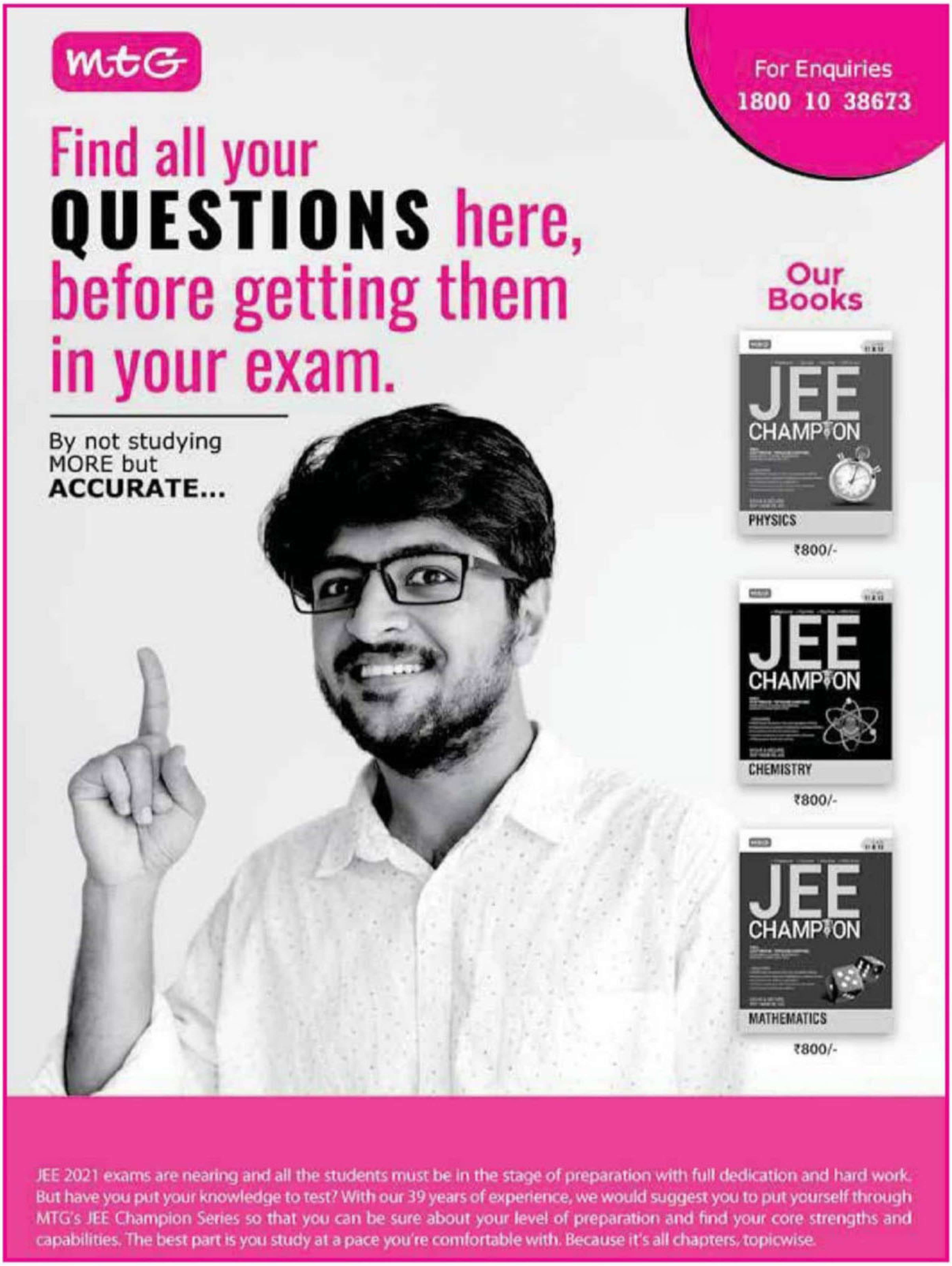
(a) length of the transverse axis $= 2\sqrt{3}$
(b) length of the conjugate axis = 8
(c) centre at (1, –2) (d) eccentricity $= \sqrt{19}$

8. The equation $2\sin\left(\frac{x}{2}\right)\cos^2 x - 2\sin\left(\frac{x}{2}\right)\sin^2 x = \cos^2 x - \sin^2 x$ has a root for which

(a) $\sin 2x = 1$ (b) $\sin 2x = -1$
(c) $\cos x = \frac{1}{2}$ (d) $\cos 2x = -\frac{1}{2}$

9. If $\int \frac{dx}{\cos(x-a)\cos(x-b)} = \frac{1}{A}[\log_e(f(x))/\log_e g(x)] + c$, then

(a) $A = \sin(a-b)$ (b) $f(x) = \sec(x-b)$
(c) $g(x) = \sec(x-a)$ (d) none of these



mtG

For Enquiries
1800 10 38673

Find all your
QUESTIONS here,
before getting them
in your exam.

By not studying
MORE but
ACCURATE...

Our
Books

JEE
CHAMPION
PHYSICS
₹800/-

JEE
CHAMPION
CHEMISTRY
₹800/-

JEE
CHAMPION
MATHEMATICS
₹800/-

JEE 2021 exams are nearing and all the students must be in the stage of preparation with full dedication and hard work. But have you put your knowledge to test? With our 39 years of experience, we would suggest you to put yourself through MTG's JEE Champion Series so that you can be sure about your level of preparation and find your core strengths and capabilities. The best part is you study at a pace you're comfortable with. Because it's all chapters, topicwise.

10. If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then

(a) f is continuous but not differentiable at $x = 0$
(b) f is differentiable at $x = 0$
(c) f is not differentiable at $x = 0$
(d) f is continuous at $x = 0$

11. In a triangle ABC, if $a = \sqrt{6}, c = \sqrt{8}$ and $A = \frac{\pi}{4}$, its area is

(a) $2-\sqrt{2}$ (b) $3-\sqrt{2}$ (c) $2+\sqrt{2}$ (d) $3+\sqrt{2}$

12. Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, \ldots\}$. Further $H_1 = \left\{z \in C : \text{Re}\, z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in C : \text{Re}\, z < -\frac{1}{2}\right\}$ where C is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

SECTION-3

This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.

13. Let $f : R^+ \to R$ be a function which satisfies $f(x) \cdot f(y) = f(xy) + 2\left(\frac{1}{x} + \frac{1}{y} + 1\right)$ for $x, y > 0$. Then possible value of $f(1/2)$ is

14. Water is dropped at the rate of 2 m^3/s into a cone of semi vertical angle 45°. If the rate at which periphery of water surface changes when the height of the water in the cone is 2 m is d, then the value of 5 d is

15. If $\sin\left\{2\left(\sin^{-1}\frac{\sqrt{5}}{3} - \cos^{-1}\frac{\sqrt{5}}{3}\right)\right\}$ is equal to $\frac{k\sqrt{5}}{81}$, then $k =$

16. If $g(x)$ is a differentiable function $g(1) \neq 0, g(-1) \neq 0$ and Rolle's theorem is not applicable to $f(x) = \frac{x^2-1}{g(x)}$ in $[-1, 1]$, then $g(x)$ has atleast k root in $(-1, 1)$, then $k =$

17. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then $16\, P(X \geq 1) =$

18. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then $\vec{r} \cdot \vec{b} =$

PAPER-2

SECTION-1

This section contains SIX (06) questions. The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE.

1. If $\int_{\pi/2}^{x} \sqrt{(3-2\sin^2 t)} + \int_{0}^{y} \cos t\, dt = 0$ and $\left(\frac{dy}{dx}\right)_{(\pi, \pi)}$ is equal to S, then value of $\sqrt{3}\, S$ is

2. If on a given base BC [where $B(0, 0)$ and $C(2, 0)$], a triangle is described such that the sum of the tangents of the base angles is 4, then the equation of the locus of the opposite vertex A is a parabola whose directrix is $y = k$. The value of $8k - 9$ is

3. The distance of the point $\hat{i} + 2\hat{j} + 3\hat{k}$ from the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 5$ measured parallel to the vector $2\hat{i} + 3\hat{j} - 6\hat{k}$ must be

4. Let $f(x) = \begin{vmatrix} \cos x & -x & 1 \\ 2\sin x & -x^2 & 2x \\ \tan x & -x & 1 \end{vmatrix}$.

The value of $\lim_{x \to 0} \frac{f'(x)}{x}$ is

5. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2 y^6$ is $\frac{1}{x^5 y^5} = \frac{p}{qx^2} + c$, then the value of $p + q$ is

6. If A is the area bounded by $y = x^{2/3} - x^{4/3}$ and x-axis, then the value of $35A/4$ is

SECTION-2

This section contains SIX (06) questions. Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).

7. Let A and B be two 3×3 matrices of real numbers, where A is symmetric and B is skew symmetric. If $(A + B)(A - B) = (A - B)(A + B)$ and $(AB)^T = (-1)^k AB$, then the possible value(s) of k is/are

(a) 2 (b) 3 (c) 5 (d) 6

8. If $\tan^{-1} y = 4 \tan^{-1} x$, then y is infinite if

(a) $x^2 = 3 + 2\sqrt{2}$ (b) $x^2 = 3 - 2\sqrt{2}$
(c) $x^4 = 6x^2 - 1$ (d) $x^4 = 6x^2 + 1$

9. The value of a, for which the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$, $a\hat{i} - 3\hat{j} + \hat{k}$

respectively are the vertices of a right angled triangle with $C=\frac{\pi}{2}$ are

(a) 1 (b) 2 (c) −1 (d) −2

10. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}=0$ if

(a) a, b, c are in A.P. (b) a, b, c are in G.P.
(c) α is a root of $ax^2+2bx+c=0$
(d) $(x-\alpha)$ is a factor of $ax^2+2bx+c$

11. If 2 and 31 appear as two terms in an A.P., then
(a) common difference of the A.P. is a rational number
(b) all the terms of the A.P. must be rational.
(c) all the terms of the A.P. must be integers.
(d) sum to any finite number of terms of the A.P. must be rational.

12. Three lines $px+qy+r=0$, $qx+ry+p=0$ and $rx+py+q=0$ are concurrent, if
(a) $p+q+r=0$
(b) $p^2+q^2+r^2=pq+qr+rp$
(c) $p^3+q^3+r^3=3pqr$
(d) none of these

SECTION-3

This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.

13. The sum of two digit even numbers which do not end with zero is

14. If area between the curves $y=xe^x$ and $y=xe^{-x}$ and the line $x=1$ is λ sq. units, then the value of $128e\lambda$ must be

15. If $\alpha=2\sin^{-1}(2/3)$ and $\beta=2\tan^{-1}9$, then $80\operatorname{cosec}^2\alpha+81\operatorname{cosec}^2\beta$ is equal to

16. The length of the common chord of two circles of radii 15 and 20, whose centres are 25 units apart, is

17. The tangents drawn from a point P to the ellipse $9x^2+16y^2=144$ make angles θ_1, θ_2 with the major axis. If $\theta_1+\theta_2=2\alpha$, then the locus of P is $x^2-2xy\cot2\alpha-y^2=\lambda$, where λ is

18. If $\int\frac{(\sqrt{x})^5}{(\sqrt{x})^7+x^6}dx=a\log\left(\frac{x^\lambda}{x^\lambda+1}\right)+C$, then $a\lambda=$

SOLUTIONS

PAPER-1

1. (b) : Let $A=(0,1)$, $B=(0,0)$, $C=(1,0)$, $D=(1,1)$
Family of circles passing through A, B is
$x^2+y^2-y+\lambda x=0$
Also, $\sqrt{1+\lambda}=2 \Rightarrow \lambda=3$

2. (b) : The line $\frac{x}{p}+\frac{y}{q}=1$ where p is a prime and q is a positive integer. Also, it passes through (4, 3).

$$\Rightarrow \quad q=\frac{3p}{p-4}=3+\frac{12}{p-4}$$

Since, q is a positive integer, $p>4$ and $(p-4)$ divides 12, there are only two such primes, $p=5$ and $p=7$.

3. (a) : Way 1 : 1 take 1, 2 take 2, 3 take 3, and 4 take 4

$$\therefore \quad P(\text{way }1)=\frac{1}{4}\times1\times1\times1=\frac{1}{4}$$

Way 2 : 1 take 2, 2 take 1, 3 take 3, 4 take 4

$$\therefore \quad P(\text{way }2)=\frac{1}{4}\times\frac{1}{3}\times1\times1=\frac{1}{12}$$

Way 3 : 1 take 2, 2 take 3, 3 take 1, 4 take 4

$$\therefore \quad P(\text{way }3)=\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times1=\frac{1}{24}$$

Way 4 : 1 take 3, 2 take 2, 3 take 1, 4 take 4

$$\therefore \quad P(\text{way }4)=\frac{1}{4}\times1\times\frac{1}{2}\times1=\frac{1}{8}$$

$$\text{Required probability}=\frac{1}{4}+\frac{1}{12}+\frac{1}{24}+\frac{1}{8}=\frac{1}{2}$$

4. (c) : $\left(\frac{t^3}{3}-\frac{8t^2}{2}+13t\right)_0^x=x\sin\left(\frac{a}{x}\right)$

$$\Rightarrow \frac{1}{3}(x^2-12x+39)=\sin\left(\frac{9}{x}\right)$$

Here $x\neq0 \Rightarrow \frac{1}{3}(x^2-12x+39)=\sin\left(\frac{9}{x}\right)$

$$\Rightarrow \frac{1}{3}(x-6)^2+1=\sin\left(\frac{a}{x}\right)$$

$$\Rightarrow \frac{a}{6}=\frac{\pi}{2} \text{ or } \frac{5\pi}{2} \text{ or } \frac{9\pi}{2}$$

$\therefore$ $a=3\pi, 15\pi, 27\pi$ (3 values)

5. (c) : $\int\frac{\sec^2x-2010}{\sin^{2010}x}dx$

$$= \int \sec^2 x(\sin x)^{-2010}dx - 2010\int \frac{1}{(\sin x)^{2010}}dx = I_1 - I_2$$

Now, applying by parts on I_1, we get

$$I_1 = \frac{\tan x}{(\sin x)^{2010}} + 2010\int \frac{\tan x \cos x}{(\sin x)^{2011}}dx$$

$$= \frac{\tan x}{(\sin x)^{2010}} + 2010\int \frac{dx}{(\sin x)^{2010}}$$

$$\Rightarrow \quad I_1 - I_2 = \frac{\tan x}{(\sin x)^{2010}} = \frac{P(x)}{(\sin x)^{2010}}$$

$$\therefore \quad P\left(\frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$$

6. (a) : $S = {}^nC_1 \sin a + {}^nC_2 \sin 2a + ... + {}^nC_n \sin na$

and $C = {}^nC_0 + {}^nC_1 \cos a + {}^nC_2 \cos 2a + ... + {}^nC_n \cos na$

$\Rightarrow \quad C + iS = 1 + {}^nC_1(\cos a + i\sin a)$

$+ {}^nC_2(\cos 2a + i\sin 2a) + ... + {}^nC_n(\cos na + i\sin na)$

$\Rightarrow \quad C + iS = (1 + z)^n$, where $z = \cos a + i\sin a$

$$= \left\{2\cos\frac{a}{2}\left(\cos\frac{a}{2} + i\sin\frac{a}{2}\right)\right\}^n$$

$$\Rightarrow \quad S = 2^n\cos^n\left(\frac{a}{2}\right)\cdot\sin\left(\frac{na}{2}\right)$$

7. (a,b,c) : $16x^2 - 3y^2 - 32x - 12y - 44 = 0$

$\Rightarrow \quad 16(x^2 - 2x) - 3(y^2 + 4y) - 44 = 0$

$\Rightarrow \quad 16\{(x-1)^2 - 1\} - 3\{(y+2)^2 - 4\} - 44 = 0$

$\Rightarrow \quad 16(x-1)^2 - 3(y+2)^2 = 48$

$$\Rightarrow \quad \frac{(x-1)^2}{(\sqrt{3})^2} - \frac{(y+2)^2}{4^2} = 1$$

Now, length of transverse axis $= 2\times\sqrt{3} = 2\sqrt{3}$

Length of conjugate axis = $2 \times 4 = 8$

Centre is given by $x - 1 = 0$ and $y + 2 = 0$, *i.e.*, $(1, -2)$

Also, $4^2 = (\sqrt{3})^2(e^2 - 1) \;\Rightarrow\; e^2 - 1 = \frac{16}{3}$

$$\therefore \quad e^2 = \frac{19}{3} \;\Rightarrow\; e = \sqrt{\frac{19}{3}}$$

8. (a,b,c,d) : $2\sin\frac{x}{2}(\cos^2 x - \sin^2 x) - (\cos^2 x - \sin^2 x) = 0$

$$\Rightarrow \quad \left(2\sin\frac{x}{2} - 1\right)(\cos^2 x - \sin^2 x) = 0$$

$$\Rightarrow \quad \left(2\sin\frac{x}{2} - 1\right)\cos 2x = 0 \;\Rightarrow\; \sin\frac{x}{2} = \frac{1}{2} \text{ or } \cos 2x = 0$$

$\Rightarrow \quad x = 60°$ and $x = \pm 45°$

9. (a,b,c) : Let $I = \int \frac{dx}{\cos(x-a)\cos(x-b)}$

$$\Rightarrow \quad I = \frac{1}{\sin(a-b)}\int \frac{\sin((x-b)-(x-a))\,dx}{\cos(x-a)\cos(x-b)}$$

$$= \frac{1}{\sin(a-b)}\int(\tan(x-b) - \tan(x-a))\,dx$$

10. (b, d) : Clearly, $f(x)$ is defined for $x > 0$

$$\text{Also } f'(0^+) = \lim_{h\to 0^+}\frac{h(\sqrt{h} - \sqrt{h+1})}{h} = -1.$$

Hence f is differentiable at $x = 0$ (see the definition of differentiability at the end point of an interval). This also shows f is continuous at $x = 0$.

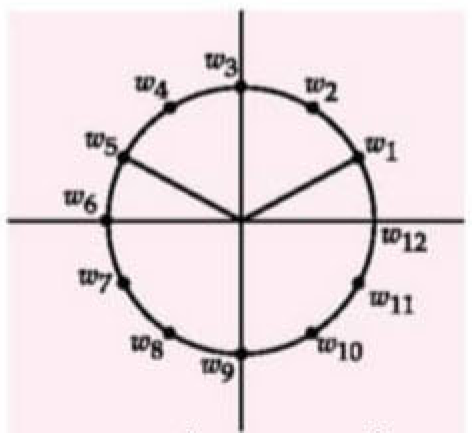
11. (a, c): $a^2 = b^2 + c^2 - 2bc\cos A = b^2 + 8 - 4b$

$$a = \sqrt{6} \Rightarrow b^2 - 4b + 2 = 0 \;\;\therefore\;\; b = 2 \pm \sqrt{2}, \Delta = \frac{1}{2}bc\sin A$$

$$\therefore \quad \Delta = \frac{1}{2}(2 \pm \sqrt{2})\sqrt{8} \times \frac{1}{\sqrt{2}} = 2 \pm \sqrt{2}$$

12. (c, d) : $w = \frac{\sqrt{3}+i}{2} = \frac{\sqrt{3}}{2} + i\frac{1}{2} = e^{i\pi/6}$

$w^n = e^{\frac{in\pi}{6}}$



$z_1 \in \{w_1, w_{11}, w_{12}\}$
$z_2 \in \{w_5, w_6, w_7\}$

For z_1, we have $\cos\frac{n\pi}{6} > \frac{1}{2}$ and

For z_2, we have $\cos\frac{n\pi}{6} < -\frac{1}{2}$

$$z_1 = \frac{\sqrt{3}+i}{2}, \frac{\sqrt{3}-i}{2}, 1$$

$$z_2 = -1, \frac{-\sqrt{3}+i}{2}, \frac{-\sqrt{3}-i}{2} \quad \therefore \quad \angle z_1Oz_2 = \frac{5\pi}{6}, \frac{2\pi}{3}$$

NOTIFICATION!

JEE Advanced 2021

The scheduled date for JEE (Advanced) 2021 is 3rd July, 2021 (Saturday)

The IIT entrance exam, JEE Advanced will be held on July 3, the Minister of Education Ramesh Pokhriyal announced through a live interaction. The exam has been deferred as a consequence of the postponement of the JEE Main exams. Just like last year, this year too the board exam criterion has been relaxed. Apart from clearing JEE Main, students also have to secure at least 75 per cent marks in the class 12 exams to be eligible. This criterion was relaxed last year and this year as well, the minister announced. Students, however, will have to pass the board exams.

As per the rule, students have to clear JEE Main every year to be eligible for the Advanced exam of the subsequent year but due to the pandemic, students who cleared JEE Main 2020 but did not appear for JEE Advanced 2020 will also be allowed to appear for Advanced 2021. This is a one-time option.

13. (4) : Putting $x = 1$ and $y = 1$, we get
$f^2(1) - f(1) - 6 = 0 \Rightarrow f(1) = 3$ or $f(1) = -2$
Now, putting $y = 1$, we get

$$f(x)\cdot f(1)=f(x)+2\left(\frac{1}{x}+2\right)=f(x)+2\left(\frac{2x+1}{x}\right)$$

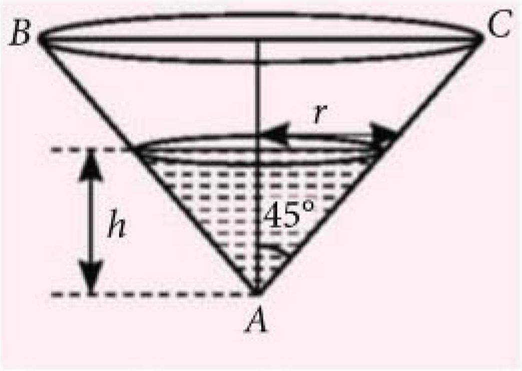
$$\Rightarrow\ f(x)[f(1)-1]=\frac{2(2x+1)}{x}\ \Rightarrow\ f(x)=\frac{2(2x+1)}{x[f(1)-1]}$$

$$\text{For } f(1)=3,\ f(x)=\frac{(2x+1)}{x} \qquad \text{...(i)}$$

$$\text{and for } f(1)=-2,\ f(x)=\frac{2(2x+1)}{-3x} \qquad \text{...(ii)}$$

$$\Rightarrow\ f\left(\frac{1}{2}\right)=4$$

14. (5) : We have,



$$\frac{dV}{dt}=2 \text{ or } \frac{d}{dt}\left(\frac{1}{3}\pi r^3\right)=2$$

$[\because r = h$ as $\theta = 45°]$

$$\Rightarrow\ \pi r^2\frac{dr}{dt}=2 \Rightarrow \frac{dr}{dt}=\frac{2}{\pi r^2} \qquad \text{...(i)}$$

Now, perimeter = $2\pi r = p$ (say)

$$\therefore\ \frac{d}{dt}(2\pi r)=2\pi\frac{dr}{dt}=2\pi\left(\frac{2}{\pi r^2}\right)=\frac{4}{r^2} \quad \text{...(ii) [Using (i)]}$$

When $h = 2$ m, $r = 2$ m

Hence, $\frac{dp}{dt}=\frac{4}{4}$ = 1 m/s.

15. (8) : Let $\alpha=\sin^{-1}\frac{\sqrt{5}}{3}$, then $\cos^{-1}\frac{\sqrt{5}}{3}=\frac{\pi}{2}-\alpha$

Hence the expression = $\sin\left\{2\left(\alpha-\frac{\pi}{2}+\alpha\right)\right\}$

= $\sin(4\alpha - \pi) = -\sin 4\alpha = -4\sin\alpha\cos\alpha(1 - 2\sin^2\alpha)$

$$=(-4)\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)\left(\frac{-1}{9}\right)=\frac{8\sqrt{5}}{81}$$

16. (1) : As $f(-1) = f(1)$ and Rolle's theorem is not applicable, then it implies $f(x)$ is either discontinuous or $f'(x)$does not exist at atleast one point in $(-1, 1)$.
$\Rightarrow$ $g(x) = 0$ at atleast one value of x in $(-1, 1)$. Thus $k = 1$

17. (15) : $np = 2$ ...(i)
$npq = 1$ ...(ii)

$$\Rightarrow\ q=\frac{1}{2},\ p=\frac{1}{2},\ n=4 \ \text{(from (i) and (ii))}$$

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$$= 1-{}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4=1-\frac{1}{16}=\frac{15}{16}$$

18. (9) : $\vec{r}\times\vec{b}=\vec{c}\times\vec{b} \Rightarrow (\vec{r}-\vec{c})\times\vec{b}=\vec{0}$

$$\Rightarrow\ \vec{r}=\vec{c}+\lambda\vec{b}$$

$$\Rightarrow\ \vec{r}\cdot\vec{a}=\vec{a}\cdot\vec{c}+\lambda\vec{a}\cdot\vec{b} \Rightarrow \vec{r}=\vec{c}-\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\vec{b}$$

$$\Rightarrow\ \vec{r}\cdot\vec{b}=\vec{b}\cdot\vec{c}-\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\left(|\vec{b}|^2\right)=1-\frac{-4}{1}(2)=9$$

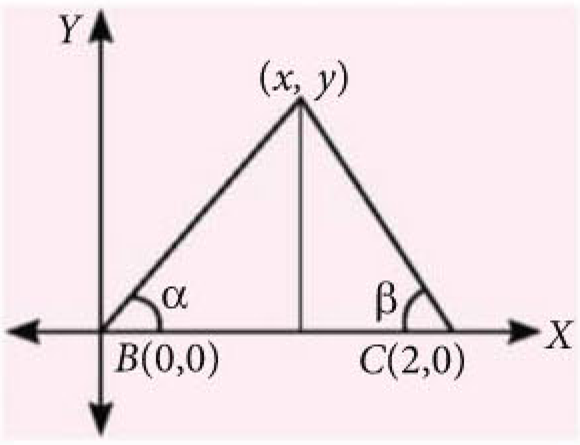
PAPER-2

1. (3) : Differentiating both sides w.r.t. x, then

$$\sqrt{(3-2\sin^2 x)}+\cos y\frac{dy}{dx}=0$$

$$\therefore\ \frac{dy}{dx}=-\frac{\sqrt{3-2\sin^2 x}}{\cos y} \quad \therefore\ \left.\frac{dy}{dx}\right|_{(\pi,\pi)}=-\frac{\sqrt{3}}{(-1)}=\sqrt{3}$$

2. (8) : Given, $\tan\alpha + \tan\beta = 4$



$$\Rightarrow\ \frac{y}{x}+\frac{y}{2-x}=4$$

$\Rightarrow$ $y = 2x(2 - x)$

$$\Rightarrow\ -\frac{y}{2}=x^2-2x$$

$= (x - 1)^2 - 1$

or $(x - 1)^2 = -\frac{1}{2}(y-2)$

$\therefore$ Directrix is $y - 2 = \frac{1}{8} \Rightarrow y=\frac{17}{8}$

3. (7) : The distance of the point $\vec{a}$ from the plane $\vec{r}\cdot\vec{n}=q$ measured in the direction of the unit vector $\hat{b}$ is $\left|\frac{q-\vec{a}\cdot\vec{n}}{\hat{b}\cdot\vec{n}}\right|$.

Here, $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{n}=\hat{i}+\hat{j}+\hat{k}$ and $q = 5$

$$\text{Also } \hat{b}=\frac{2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{(2)^2+(3)^2+(-6)^2}}=\frac{2\hat{i}+3\hat{j}-6\hat{k}}{7}$$

$\therefore$ The required distance

$$=\left|\frac{5-(\hat{i}+2\hat{j}+3\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})}{\frac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})}\right|=\left|\frac{5-(1+2+3)}{\frac{1}{7}(2+3-6)}\right|=7$$

Monthly Test Drive-9 CLASS XII ANSWER KEY

1. (b)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (c)
6. (c)	**7.** (c)	**8.** (a,c)	**9.** (a,c)	**10.** (a,d)
11. (b,d)	**12.** (a,b)	**13.** (a,c,d)	**14.** (d)	**15.** (b)
16. (a)	**17.** (7)	**18.** (3)	**19.** (5)	**20.** (1)

4. (2) : Differentiating $-f(x)$ w.r.t. x, we get

$$-f'(x)=\begin{vmatrix}-\sin x & x & 1\\ 2\cos x & x^2 & 2x\\ \sec^2 x & x & 1\end{vmatrix}+\begin{vmatrix}\cos x & 1 & 1\\ 2\sin x & 2x & 2x\\ \tan x & 1 & 1\end{vmatrix}+\begin{vmatrix}\cos x & x & 0\\ 2\sin x & x^2 & 2\\ \tan x & x & 0\end{vmatrix}$$

$$\Rightarrow\quad -\frac{f'(x)}{x}=\begin{vmatrix}-\sin x & 1 & 1\\ 2\cos x & x & 2x\\ \sec^2 x & 1 & 1\end{vmatrix}+\begin{vmatrix}\cos x & 1 & 0\\ 2\sin x & 2x & 2\\ \tan x & 1 & 0\end{vmatrix}$$

[∵ The second determinant is zero]

$$\Rightarrow\quad -\lim_{x\to 0}\frac{f'(x)}{x}=\begin{vmatrix}0 & 1 & 1\\ 2 & 0 & 0\\ 1 & 1 & 1\end{vmatrix}+\begin{vmatrix}1 & 1 & 0\\ 0 & 0 & 2\\ 0 & 1 & 0\end{vmatrix}=-2$$

5. (7) : $\frac{dy}{dx}+\frac{y}{x}=x^2y^6 \Rightarrow \frac{1}{y^6}\frac{dy}{dx}+\frac{1}{xy^5}=x^2$

Put $\frac{1}{y^5}=z \Rightarrow \frac{-5}{y^6}\frac{dy}{dx}=\frac{dz}{dx}$

$\therefore\ -\frac{1}{5}\left(\frac{dz}{dx}\right)+\frac{z}{x}=x^2 \Rightarrow \text{I.F.}=\frac{1}{x^5}$

Hence solution is $\frac{1}{x^5y^5}=\frac{5}{2x^2}+c$

6. (3) : We have $y = f(x) = x^{2/3} - x^{4/3}$

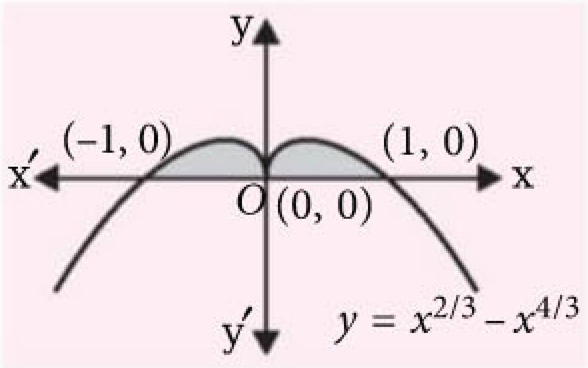
It meets the x-axis at $x = 0, \pm 1$.

Also, function is even.

$\therefore$ Required area,

$$A=2\left[\int_0^1 (x^{2/3}-x^{4/3})\,dx\right]$$

$$=2\left[\frac{3x^{5/3}}{5}-\frac{3x^{7/3}}{7}\right]_0^1=\frac{12}{35}$$



7. (b, c) : Since A is symmetric, $A^T = A$.

Since B is skew-symmetric, $B^T = -B$.

Given, $(A + B)(A - B) = (A - B)(A + B)$

$\Rightarrow A^2 - AB + BA - B^2 = A^2 + AB - BA - B^2$

$\Rightarrow 2BA = 2AB \Rightarrow AB = BA$

Also, $(AB)^T = (-1)^k AB$

$\Rightarrow B^TA^T = (-1)^k AB \Rightarrow -BA = (-1)^k AB$

$\Rightarrow -AB = (-1)^k AB \Rightarrow AB = (-1)^{k+1} AB$

$\therefore\ k$ = odd number

8. (a, b, c) : If we put $x = \tan\theta$, the given equality becomes $\tan^{-1} y = 4\theta$.

$$\Rightarrow\quad y=\tan 4\theta=\frac{2\tan 2\theta}{1-\tan^2 2\theta}=\frac{2\left[\frac{2\tan\theta}{1-\tan^2\theta}\right]}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}$$

$$=\frac{2\times 2x(1-x^2)}{(1-x^2)^2-4x^2}=\frac{4x(1-x^2)}{1-6x^2+x^4}$$

Now, y is infinite if $x^4 - 6x^2 + 1 = 0$

$$\Rightarrow\quad x^2=\frac{6\pm\sqrt{36-4}}{2}=3\pm 2\sqrt{2}$$

9. (a, b) : $\overrightarrow{BC}=(a\hat{i}-3\hat{j}+\hat{k})-(\hat{i}-3\hat{j}-5\hat{k})$

$=(a-1)\hat{i}+6\hat{k}$

$\overrightarrow{CA}=(2\hat{i}-\hat{j}+\hat{k})-(a\hat{i}-3\hat{j}+\hat{k})=(2-a)\hat{i}+2\hat{j}$

$\overrightarrow{BC}\cdot\overrightarrow{CA}=0 \Rightarrow (a-1)(2-a)=0$

$\therefore\ a = 1, 2.$

10. (b, c, d) : $\begin{vmatrix}a & b & a\alpha+b\\ b & c & b\alpha+c\\ a\alpha+b & b\alpha+c & 0\end{vmatrix}$

$$=\begin{vmatrix}a & b & 0\\ b & c & 0\\ a\alpha+b & b\alpha+c & -(a\alpha^2+2b\alpha+c)\end{vmatrix}$$

[Applying $C_3 \to C_3 - \alpha C_1 - C_2$]

$= (b^2 - ca)(a\alpha^2 + 2b\alpha + c) = 0$

if a, b, c are in G.P. or α is a root of $ax^2 + 2bx + c = 0$.

11. (a, b, d) : Let d be the common difference and $a_m = 2$, $a_n = 31$, then $(n - m)d = 29 \Rightarrow d = \frac{29}{n-m}$ which is rational.

Also $a_r - a_m = (r - m)d$

$\Rightarrow\ a_r$ is rational.

As each term is rational, sum to any finite number of terms must be rational.

12. (a, c) : $\begin{vmatrix}p & q & r\\ q & r & p\\ r & p & q\end{vmatrix}=0$

$\Rightarrow\ p(rq - p^2) - q(q^2 - rp) + r(pq - r^2) = 0$

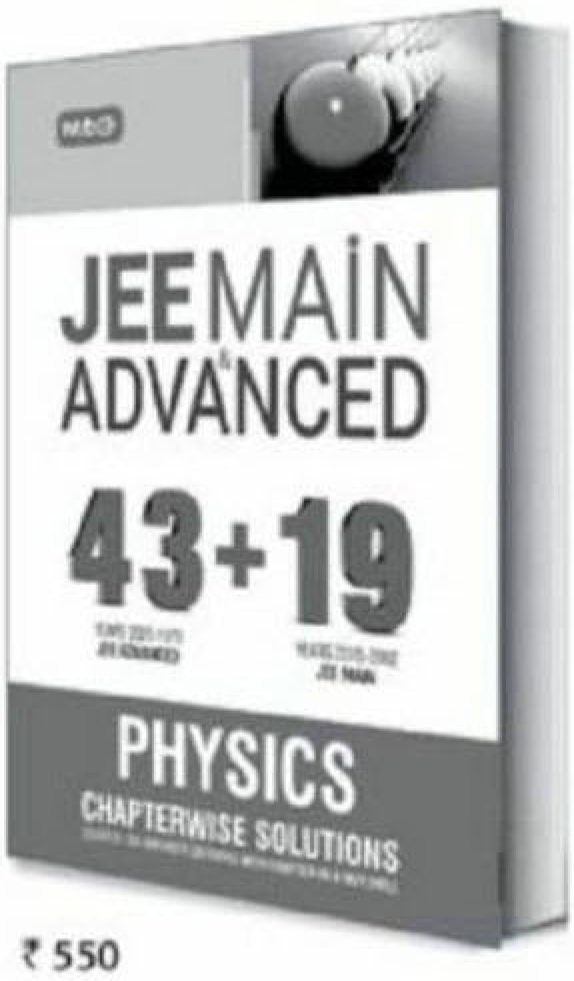
Monthly Test Drive-9 CLASS XI ANSWER KEY

1.	(c)	**2.**	(b)	**3.**	(c)	**4.**	(c)	**5.**	(d)
6.	(b)	**7.**	(a,c)	**8.**	(c,d)	**9.**	(a,b,d)	**10.**	(a,b,c)
11.	(a,b,c,d)	**12.**	(c,d)	**13.**	(a)	**14.**	(b)	**15.**	(d)
16.	(c)	**17.**	(3)	**18.**	(3)	**19.**	(2)	**20.**	(2)

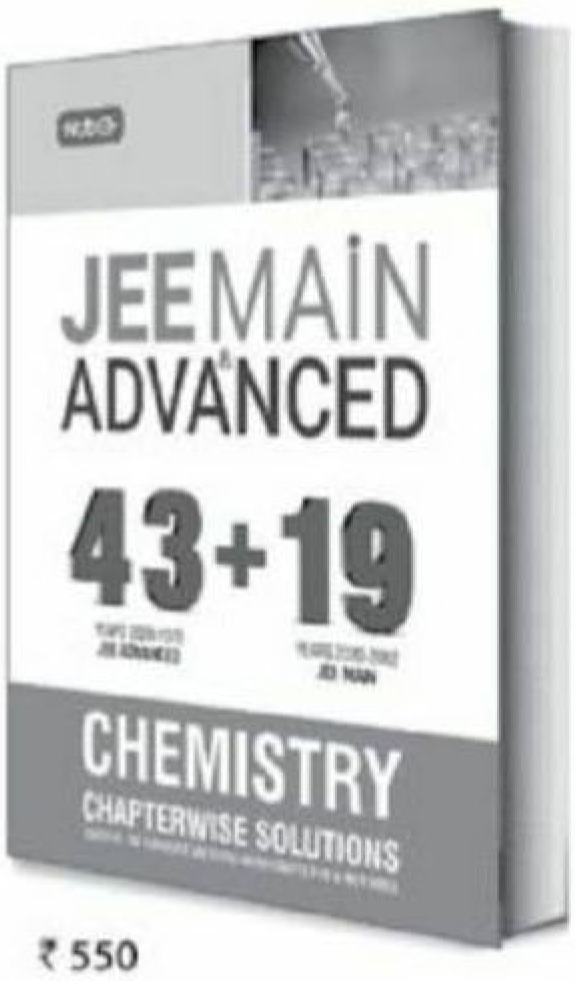
MtG

Some of the best lessons are learnt from history!

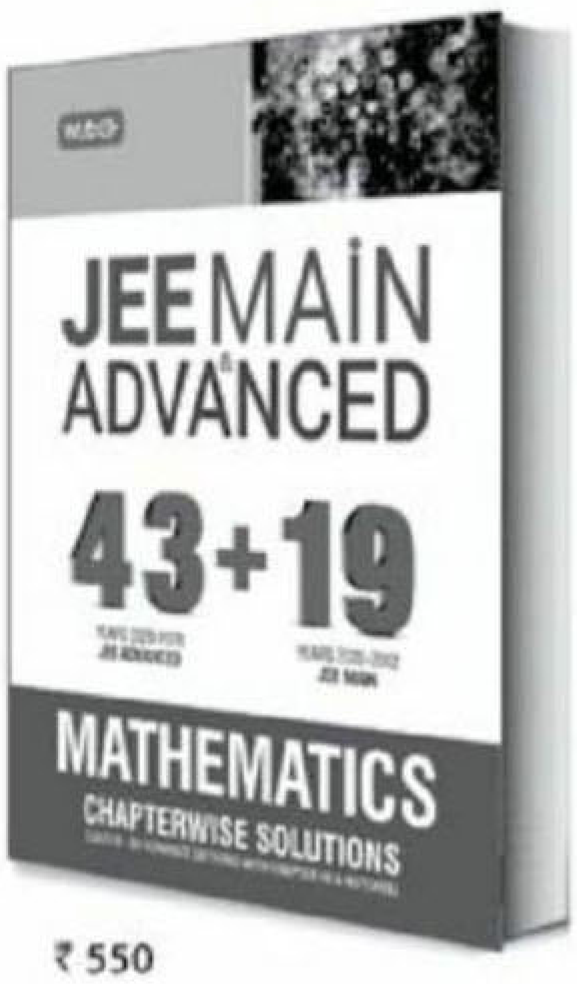
Introducing Chapterwise Solutions to JEE.
Crack the past to see if you will last.


JEE MAIN & ADVANCED
43+19
PHYSICS
CHAPTERWISE SOLUTIONS

₹ 550


JEE MAIN & ADVANCED
43+19
CHEMISTRY
CHAPTERWISE SOLUTIONS

₹ 550


JEE MAIN & ADVANCED
43+19
MATHEMATICS
CHAPTERWISE SOLUTIONS

₹ 550

Tune. Fine tune. Reach the peak of your readiness for JEE with MTG's 43+19 Years Chapterwise Solutions. It is undoubtedly the most comprehensive 'real' question bank, complete with detailed solutions by experts.

Studies have shown that successful JEE aspirants begin by familiarising themselves with the problems that have appeared in past JEEs as early as 2 years in advance.

Making it one of the key ingredients for their success. How about you then?
Get 43+19 Years Chapterwise Solutions to start your rehearsals early.

MtG
Learning Media

Available at all leading book shops throughout the country.
For more information or for help in placing your order:
Visit **www.mtg.in** to buy online!
Call 0124-6601200 or e-mail:info@mtg.in

or $p^3 + q^3 + r^3 - 3pqr = 0$

Now, $p^3 + q^3 + r^3 - 3pqr$

$= (p + q + r)(p^2 + q^2 + r^2 - pq - qr - rp)$

$\therefore \quad p + q + r = 0$

[$\because\ p^2 + q^2 + r^2 - pq - qr - rp = 0$ only if $p = q = r$. In this case all the three lines are same, $x + y + 1 = 0$.]

13. (1980) : Required sum

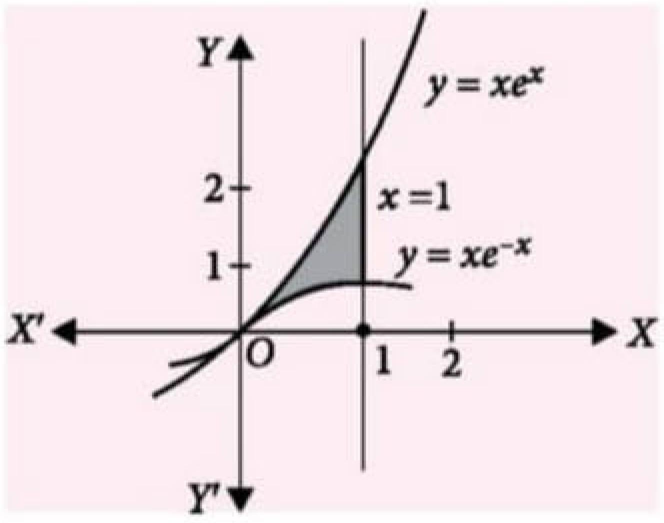
$= (12 + 14 + 16 + \ldots + 98) - (20 + 30 + 40 + \ldots + 90)$

$$= \frac{44}{2}(12+98) - \frac{8}{2}(20+90)$$

$= 22(110) - 4(110) = (110)(18) = 1980$

14. (256) : Required area $(\lambda) = \int_0^1 (xe^x - xe^{-x})dx$

$$= [x(e^x + e^{-x}) - (e^x - e^{-x})]_0^1$$



$$= [(e + e^{-1}) - (e - e^{-1})] - [0 - (1 - 1)] = \frac{2}{e}$$

$$\therefore \quad 128e\lambda = 128e \times \frac{2}{e} = 256$$

15. (1762) : Here, $\alpha = 2\sin^{-1}\left(\frac{2}{3}\right)$

$$= \sin^{-1}\left[2 \times \frac{2}{3}\sqrt{1 - \frac{4}{9}}\right] = \sin^{-1}\frac{4\sqrt{5}}{9}$$

$\Rightarrow \quad 81\sin^2\alpha = 80 \Rightarrow 80\operatorname{cosec}^2\alpha = 81$

$$\text{Also, } \beta = 2\tan^{-1} 9 = \pi - \sin^{-1}\frac{2 \times 9}{1 + 81} = \pi - \sin^{-1}\frac{9}{41}$$

$$\Rightarrow \quad \sin\beta = \frac{9}{41} \quad \Rightarrow 81\operatorname{cosec}^2\beta = 1681$$

$\therefore \quad 80\operatorname{cosec}^2\alpha + 81\operatorname{cosec}^2\beta = 1681 + 81 = 1762$

16. (24) : Let C_1 and C_2 be the centres of the two circles of radii r and R respectively.

Then, $C_1C_2 = 25$, $r = 15$ and $R = 20$ [Given]

We observe that $C_1C_2^2 = r^2 + R^2$

Thus, the two circles, intersect orthogonally.

$\therefore$ Length of the common chord

$$= \frac{2rR}{\sqrt{r^2 + R^2}} = \frac{2 \times 15 \times 20}{25} = 24$$

17. (7) : The tangent with slope m to

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \text{ is } y = mx \pm \sqrt{16m^2 + 9}$$

$\Rightarrow \quad (mx - y)^2 = 16m^2 + 9$

$\Rightarrow \quad (16 - x^2)m^2 + 2xym + 9 - y^2 = 0$

$$\tan\theta_1 + \tan\theta_2 = \frac{-2xy}{16 - x^2}, \quad \tan\theta_1 \tan\theta_2 = \frac{9 - y^2}{16 - x^2}$$

$$\text{Now, } \theta_1 + \theta_2 = 2\alpha \Rightarrow \tan 2\alpha = \frac{-2xy}{7 - x^2 + y^2}$$

$\Rightarrow \quad 7 - x^2 + y^2 + 2xy\cot 2\alpha = 0$

$\Rightarrow \quad x^2 - 2xy\cot 2\alpha - y^2 = 7$

$\therefore \quad \lambda = 7$

18. (1) : Let $I = \int \frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6}dx$

$$= \int \frac{dx}{(\sqrt{x})^2 + (\sqrt{x})^7} = \int \frac{1}{(\sqrt{x})^7\left(1 + \frac{1}{(\sqrt{x})^5}\right)}dx$$

Let $1 + \frac{1}{(\sqrt{x})^5} = t$. Then $-\frac{5}{2}x^{-7/2}\,dx = dt$

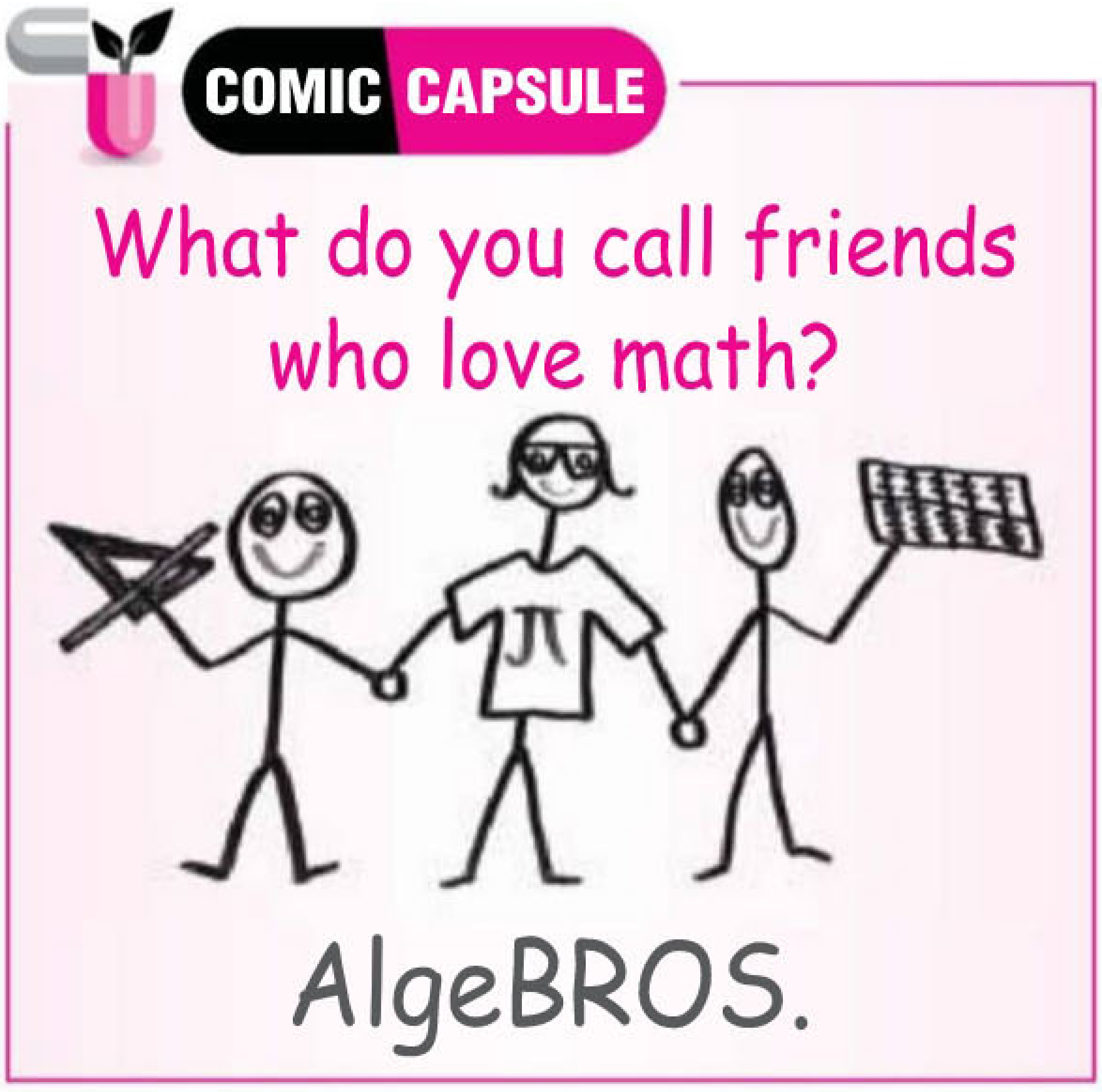
$$\therefore \quad I = -\frac{2}{5}\int\frac{dt}{t} = -\frac{2}{5}\log|t| + C = \frac{2}{5}\log\left(\frac{x^{5/2}}{1 + x^{5/2}}\right) + C$$

On comparing $a\log\left(\frac{x^\lambda}{x^\lambda + 1}\right) + C$ with

$\frac{2}{5}\log\left(\frac{x^{5/2}}{1 + x^{5/2}}\right) + C$, we get $a = \frac{2}{5}$ and $\lambda = \frac{5}{2}$

$\therefore \quad a\lambda = 1.$

◆◆


COMIC CAPSULE
What do you call friends who love math?
π
AlgeBROS.

OLYMPIAD CORNER

1. For non-negative reals x, y and z satisfying $x + y + z = 1$, show that $\left(\frac{1}{x}+1\right)\left(\frac{1}{y}+1\right)\left(\frac{1}{z}+1\right) \geq 64$.
2. Consider the infinite sum $S = \frac{a_0}{10^0}+\frac{a_1}{10^2}+\frac{a_2}{10^4}+\frac{a_3}{10^6}+\ldots,$ where the sequence $\{a_n\}$ is defined by $a_0 = a_1 = 1$, and the recurrence relation $a_n = 20a_{n-1} + 12a_{n-2}$ for all positive integers $n \geq 2$. If $\sqrt{S}$ can be expressed in the form $\frac{a}{\sqrt{b}}$, where a and b are relatively prime positive integers, determine the ordered pair (a, b).
3. Let S be the sum of the elements of the set $\{1, 2, 3, \ldots, (2p)^n - 1\}$. Let T be the sum of the elements of this set whose representation in base $2p$ consists only of digits from 0 to $p - 1$. Prove that $2^n \times \frac{T}{S} = (p-1)/(2p-1)$.
4. Let p be a prime. Find all solutions in positive integers of the equation : $\frac{2}{a}+\frac{3}{b}=\frac{5}{p}$.

SOLUTIONS

1. Expanding, we first must show

$$1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}+\frac{1}{xyz} \geq 64$$

By A.M.-G.M. inequality,

$$xyz \leq \left(\frac{x+y+z}{3}\right)^3 = \frac{1}{27} \text{ so that } \frac{1}{xyz} \geq 27.$$

Therefore,

$$1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}+\frac{1}{xyz}$$

$$\geq 1+\frac{3}{\sqrt[3]{xyz}}+\frac{3}{\sqrt[3]{x^2y^2z^2}}+\frac{1}{xyz}$$

$$= \left(1+\frac{1}{\sqrt[3]{xyz}}\right)^3 \geq 4^3 = 64$$

2. We have

$$S-\frac{20S}{10^2}-\frac{12S}{10^4} = \left(\frac{a_0}{10^0}+\frac{a_1}{10^2}+\frac{a_2}{10^4}+\frac{a_3}{10^6}+\ldots.\right)$$

$$-\left(\frac{20a_0}{10^2}+\frac{20a_1}{10^4}+\frac{20a_2}{10^6}+\frac{20a_3}{10^8}+\ldots.\right)$$

$$-\left(\frac{12a_0}{10^4}+\frac{12a_1}{10^6}+\frac{12a_2}{10^8}+\frac{12a_3}{10^{10}}+\ldots.\right)$$

$$= \frac{a_0}{10^0}+\frac{a_1}{10^2}-\frac{20a_0}{10^2}+\frac{a_2-20a_1-12a_0}{10^4}$$

$$+\frac{a_3-20a_2-12a_1}{10^6}+\frac{a_4-20a_3-12a_2}{10^8}+\ldots.$$

Since, $a_n - 20a_{n-1} - 12a_{n-2} = 0$ for all positive integers $n \geq 2$, we have

$$S-\frac{20S}{10^2}-\frac{12S}{10^4} = \frac{a_0}{10^0}+\frac{a_1}{10^2}-\frac{20a_0}{10^2},$$

and substituting in $a_0 = a_1 = 1$, we have

$$\frac{7988S}{10000} = \frac{81}{100}, \text{ so } S = \frac{2025}{1997}.$$

Hence, $\sqrt{S} = \frac{45}{\sqrt{1997}}$, and so the desired ordered pair is $(a, b) = (45, 1997)$.

3. We have $S = \frac{[(2p)^n - 1](2p)^n}{2}$. Let R denote the set of numbers that have at most n digits in base $2p$ and contain only the digits from 0 to $p - 1$. Thus T is the sum of the elements of R. For example, when $p = 2$ and $n = 3$, we have $R = \{0, 1, 10, 11, 100, 101, 110, 111\}$ and $T = 1110_4 = 84$. Note R will have p^n elements, because each of the n digits (including leading zeros) can be any one of p different numbers. Since each number in $\{0, 1, \ldots, p - 1\}$ appears the same number of times as a digit in R, there will be exactly p^{n-1} elements in R that have k as their t^{th} digit, for $k = 0, 1, \ldots,$

$p - 1$ and $t = 1, 2, ..., n$. Thus,

$$T=\sum_{i=0}^{n-1}(2p)^i\cdot[p^{n-1}\times 0+p^{n-1}\times 1+\ldots+p^{n-1}\times(p-1)]$$

$$=\sum_{i=0}^{n-1}(2p)^i\cdot p^{n-1}\cdot\frac{p(p-1)}{2}=\frac{p^n(p-1)}{2}\cdot\sum_{i=0}^{n-1}(2p)^i$$

$$=\frac{p^n(p-1)}{2}\cdot\frac{(2p)^n-1}{2}$$

Hence, $$2^n\times\frac{T}{S}=\frac{2^n\cdot\dfrac{p^n(p-1)}{2}\cdot\dfrac{(2p)^n-1}{2p-1}}{\dfrac{[(2p)^n-1](2p)^n}{2}}=\frac{p-1}{2p-1}$$

as required. This can also be solved using induction.

4. We have, $p(3a + 2b) = 5ab$; hence there are three cases to consider.

First case : $p = 5$. Then we get

$3a + 2b = ab$ or $(a - 2)(b - 3) = 6 = 1\cdot 6 = 2\cdot 3$

Hence, all pairs of positive integers (a, b) are $(3, 9)$, $(4, 6)$, $(5, 5)$ and $(8, 4)$.

Second case : p divides a. Let $a = a_1p$.

Hence, $3a_1p = b(5a_1 - 2)$.

Then p divides either b or $5a_1 - 2$. If $b = b_1p$, then $3a_1 = b_1(5a_1 - 2)$ or $(5a_1 - 2)(5b_1 - 3) = 6$, which has only one solution $(a_1, b_1) = (1, 1)$, when $(a, b) = (p, p)$. Otherwise, $5a_1 - 2 = a_2p$. Therefore,

$$3\frac{a_2p+2}{5}=ba_2 \text{ or } 3a_2p+6=5ba_2.$$

Hence, a_2 divides 6.

If $a_2 = 1$, then $(a, b) = \left(\frac{p(p+2)}{5}, \frac{3(p+2)}{5}\right)$, but only if $p \equiv 3 \pmod 5$.

If $a_2 = 2$, then $(a, b) = \left(\frac{2p(p+1)}{5}, \frac{3(p+1)}{5}\right)$, but only if $p \equiv 4 \pmod 5$.

If $a_2 = 3$, then $(a, b) = \left(\frac{p(3p+2)}{5}, \frac{3p+2}{5}\right)$, but only if $p \equiv 1 \pmod 5$.

If $a_2 = 6$, then $(a, b) = \left(\frac{2p(3p+1)}{5}, \frac{3p+1}{5}\right)$, but only if $p \equiv 3 \pmod 5$.

Third case : p divides b. Let $b = b_1p$.

Hence, $2b_1p = a(5b_1 - 3)$

Then p divides either a or $5b_1 - 3$. If p divides a, then we have the same case (p divides a and b) as already considered above. Again (p, p) is the (only) solution. Otherwise, $5b_1 - 3 = b_2p$. Therefore,

$$2\frac{b_2p+3}{5}=ab_2 \text{ or } 2b_2p+6=5ab_2.$$

If $b_2 = 1$, then $(a, b) = \left(\frac{2(p+3)}{5}, \frac{p(p+3)}{5}\right)$, but only if $p \equiv 2 \pmod 5$.

If $b_2 = 2$, then $(a, b) = \left(\frac{2p+3}{5}, \frac{p(2p+3)}{5}\right)$, but only if $p \equiv 1 \pmod 5$.

If $b_2 = 3$, then $(a, b) = \left(\frac{2(p+1)}{5}, \frac{3p(p+1)}{5}\right)$, but only if $p \equiv 4 \pmod 5$.

If $b_2 = 6$, then $(a, b) = \left(\frac{2p+1}{5}, \frac{3p(2p+1)}{5}\right)$, but only if $p \equiv 2 \pmod 5$.

[Note that if $p = 2$ or 3, this generates only two distinct solutions : (2, 2), (1, 6) for $p = 2$ and (3,3), (12, 2) for $p = 3$. If $p > 5$, then the three solutions are all distinct.]

◆

SAMURAI SUDOKU

ANSWER - JANUARY 2021

5	8	4	2	7	6	1	3	9				6	7	3	1	8	2	4	5	9
2	6	3	1	4	9	5	8	7				4	8	5	6	3	9	7	2	1
7	1	9	8	5	3	2	4	6				1	9	2	7	4	5	8	6	3
4	3	7	6	1	2	9	5	8				2	5	8	9	1	6	3	7	4
1	2	6	9	8	5	3	7	4				9	4	6	3	2	7	1	8	5
8	9	5	7	3	4	6	2	1				3	1	7	8	5	4	2	9	6
9	7	8	3	2	1	4	6	5	3	7	1	8	2	9	4	6	3	5	1	7
6	4	2	5	9	8	7	1	3	9	2	8	5	6	4	2	7	1	9	3	8
3	5	1	4	6	7	8	9	2	5	4	6	7	3	1	5	9	8	6	4	2
						6	7	8	1	3	9	2	4	5						
						9	5	4	7	8	2	6	1	3						
						3	2	1	6	5	4	9	8	7						
3	4	1	6	9	2	5	8	7	4	6	3	1	9	2	3	6	4	5	8	7
8	5	6	4	1	7	2	3	9	8	1	7	4	5	6	7	8	9	3	2	1
2	9	7	3	5	8	1	4	6	2	9	5	3	7	8	5	2	1	4	9	6
1	3	5	7	2	4	9	6	8				2	1	7	9	4	8	6	3	5
7	8	4	9	6	5	3	1	2				9	6	3	1	7	5	2	4	8
9	6	2	8	3	1	7	5	4				5	8	4	6	3	2	1	7	9
4	2	3	5	7	6	8	9	1				8	3	1	4	5	7	9	6	2
6	7	9	1	8	3	4	2	5				7	4	5	2	9	6	8	1	3
5	1	8	2	4	9	6	7	3				6	2	9	8	1	3	7	5	4

YOU ASK
WE ANSWER

Do you have a question that you just can't get answered? Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If $f(x)$ is a polynomial of degree <3, prove that

$$\begin{vmatrix} 1 & a & \dfrac{f(a)}{x-a} \\ 1 & b & \dfrac{f(b)}{x-b} \\ 1 & c & \dfrac{f(c)}{x-c} \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \frac{f(x)}{(x-a)(x-b)(x-c)}$$

(Ankita, U.P.)

Ans. $\dfrac{f(x)}{(x-a)(x-b)(x-c)} = \dfrac{A}{(x-a)} + \dfrac{B}{(x-b)} + \dfrac{C}{(x-c)}$

[By partial fractions]

i.e., $f(x) = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$ ...(i)

Putting $x = a$ in equation (i), we have

$f(a) = A(a-b)(a-c) \Rightarrow A = \dfrac{f(a)}{(a-b)(a-c)}$

Similarly, putting $x = b$, c in equation (i) respectively, we have $B = \dfrac{f(b)}{(b-a)(b-c)}$ and $C = \dfrac{f(c)}{(c-a)(c-b)}$

Hence, we have $\dfrac{f(x)}{(x-a)(x-b)(x-c)}$

$$= \frac{(c-b)\dfrac{f(a)}{x-a} - (c-a)\dfrac{f(b)}{(x-b)} + (b-a)\dfrac{f(c)}{(x-c)}}{(a-b)(b-c)(c-a)}$$

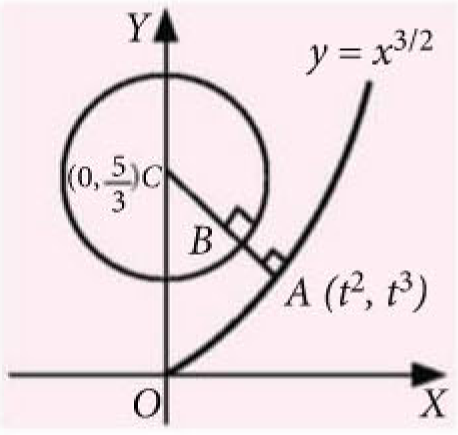
$$= \begin{vmatrix} 1 & a & \dfrac{f(a)}{x-a} \\ 1 & b & \dfrac{f(b)}{x-b} \\ 1 & c & \dfrac{f(c)}{x-c} \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$, which is the desired result.

2. Find the shortest distance between the curves $y^2 = x^3$ and $9x^2 + 9y^2 - 30y + 16 = 0$.

(Indrani Mehta, Delhi)

Ans. We have $9x^2 + 9y^2 - 30y + 16 = 0$, which is a circle having centre $\equiv (0, 5/3)$ and radius $= \sqrt{\left(\dfrac{5}{3}\right)^2 - \dfrac{16}{9}} = 1$.

Let us choose any point on the curve $y^2 = x^3$ as $A(t^2, t^3)$. If B is the point on the circle and nearest to A, then $AB = AC -$ radius



$$= \sqrt{(t^2-0)^2 + \left(t^3 - \frac{5}{3}\right)^2} - 1$$

Let $f(t) = t^4 + \left(t^3 - \dfrac{5}{3}\right)^2$

$$\Rightarrow f'(t) = 4t^3 + 2\left(t^3 - \frac{5}{3}\right)\cdot 3t^2 = 2t^2\left[2t + 3\left(t^3 - \frac{5}{3}\right)\right]$$

$= 2t^2(3t^3 + 2t - 5) = 2t^2(t-1)(3t^2 + 3t + 5)$.

The value of t at which AB attains minima is given by the equation $f'(t) = 0$ gives $t = 0, 1$.

But $t = 0$ is not a point of extrema, since $f'(t)$ does not change sign in the neighbourhood of $t = 0$.

However, $t = 1$ is a point of minima, since $f'(1^-) < 0$ and $f'(1^+) > 0$.

Hence, the minimum value of AB is

$$\sqrt{1 + \left(1 - \frac{5}{3}\right)^2} - 1 \simeq 0.2.$$

3. Show that $\displaystyle\int_b^a \frac{x^{n-1}[(n-2)x^2 + (n-1)(a+b)x + nab]}{(x+a)^2(x+b)^2}\,dx = \frac{b^{n-1} - a^{n-1}}{2(a+b)}$.

(Abhinav, Gujarat)

Ans. Let $I = \displaystyle\int_b^a \frac{x^{n-1}[(n-2)x^2 + (n-1)(a+b)x + nab]}{(x+a)^2(x+b)^2}\,dx$

$$= \int_a^b \frac{x^{n-1}[n\{x^2 + (a+b)x + ab\} - \{2x^2 + (a+b)x\}]}{(x+a)^2(x+b)^2}\,dx$$

$$= \int_a^b \frac{nx^{n-1}(x+a)(x+b) - x^n\{2x + (a+b)\}}{(x+a)^2(x+b)^2}\,dx$$

$$= \int_a^b d\left\{\frac{x^n}{(x+a)(x+b)}\right\} = \left[\frac{x^n}{(x+a)(x+b)}\right]_a^b$$

$$= \frac{b^n}{2b(a+b)} - \frac{a^n}{2a(a+b)} = \frac{b^{n-1} - a^{n-1}}{2(a+b)}.$$

❖ ❖

TOP 5 Rules to Clear JEE Main 2021 in One Go

NTA released JEE Main 2021 February exam schedule and now it's time to buckle up for JEE Main 2021 preparation. Every year JEE Main is conducted by NTA to provide admissions in various IITs, NITs, CFTIs and other renowned institutions.

This year there is a big news, Education ministry decided to conduct JEE Main 2021 exam 4 times; starting from February, March, April and May. It is an opportunity for all the JEE Main 2021 candidates to clear it by improving their score attempt after attempt. JEE candidates have four attempts in one year to achieve their dreams. The candidates will thus benefit in the following ways:

1. This will give multiple opportunities to the candidates to improve their scores in the examination if they fail to give their best in the first attempt without wasting their whole academic year.
2. In first attempt, the students will get a first-hand experience of taking an examination and will know their mistakes which they can improve while attempting for the next time.
3. This will reduce the chances of dropping a year and droppers would not have to waste a full year.
4. If anyone missed the examination due to reasons beyond control, then he/she will not have to wait for one full year.

Though the opportunities are multiple, competition is tough. Every year more than 10 Lakhs students sit for JEE Main. To help you plan better we are here sharing the top 5 rules to clear JEE Main 2021 exam in one Go.

1 Set a Time Table to Cover What's Important

Always make a clear plan. Timetable is a prerequisite. Calculate the time left and work that need to be done. Many a times you have to rush against the clock. So, divide your time equally for Conceptual reading, Question Practice, Revision and practice with Previous years papers in all three subjects. Then, a preparation will be complete in every aspect. 30 days crash course of MTG is quite helpful in make a perfect time table.

2 Study as Per Weightage

Simple studies will not give fruitful result. Create a blue print of exam syllabus and keep it handy. In the last month of preparation, no one can mug up the entire XI & XII syllabus. Find out the high weightage chapters and cover them first and then, move on to low weightage topics. To score high in JEE Main exam, cover the important ones first.

3 Practice, Practice and More Practice

Practices is the most important thing when it comes to succeeding in any field. Similarly, JEE Main is the field where you need to be quick and smart. Practice as many questions as you can after each chapter/concept. It is highly recommended to solve previous year papers to clear the idea about the type of questions asked. NTA JEE Main 2021 has all the 16 papers of JEE Main 2020 January and September. You can practice them online too. Solve these papers and analyse your studies, prepare accordingly.

4 Use Tricks to Increase Calculation Speed

Increase calculation speed by setting a timer and solve sample papers of JEE Main Exam. Check and analyse the results, where you stand. Practice until you are able to solve complete paper in the prescribed time. You may refer to MTG JEE Main Book Series as it contains short cut methods.

5 Join JEE Main Online Test Series

It is must to join JEE Main Online test series as you have to give JEE Main 2021 exam online. Mostly students' study but don't practice online exam format and get puzzled while giving main exam. In actual scenario you are going to give it online. So, it is important to practice online test series to avoid errors in the main exam. You can join MTG JEE Main Online Test Series to boost your performance.

These are the Top 5 Rules to Clear JEE Main 2021 Exam in One Go. If you follow the right path with dedication and correct study material then you can not only clear JEE Main exam but can also score a good rank in it.

For more such articles visit ***https://blog.pcmbtoday.com/***

CONCEPT BOOSTER

This column is aimed for students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

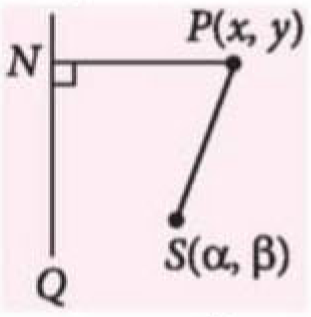
***ALOK KUMAR**

Parabola

INTRODUCTION

- A conic section or conic is the locus of a point, which moves such that its distance from a fixed point is in a constant ratio to its distance from a fixed straight line, not passing through the fixed point. The fixed point is called the focus. The fixed straight line is called the directrix. The constant ratio is called eccentricity which is denoted by e.
- When $e = 1$, the conic is a parabola. When $e < 1$, the conic is an ellipse. When $e > 1$, the conic is a hyperbola.
- The line of symmetry of the conic section is called its axis.
- A point of intersection of a conic with its axis is called vertex.
- A chord passes through the focus and perpendicular to the axis is called latus rectum.

GENERAL EQUATION OF CONIC



- Let $S(\alpha, \beta)$ be the focus and $Ax + By + C = 0$ be the equation of the directrix QN.
  Let $P(x, y)$ be any point on it and let $PN \perp QN$, then by definition

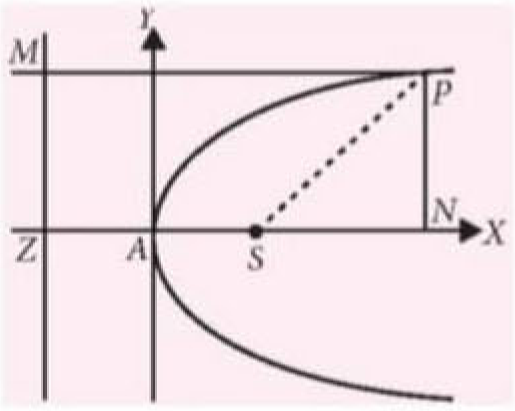
$$\frac{SP}{PN} = e \Rightarrow SP^2 = e^2 \cdot PN^2$$

$$\Rightarrow (x-\alpha)^2 + (y-\beta)^2 = e^2\left(\frac{Ax+By+C}{\sqrt{A^2+B^2}}\right)^2$$

which on simplification takes the form
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$,
which is the general equation of second degree. This will represent

(i) a pair of straight lines if $\Delta = 0$, where $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$, $h^2 \geq ab$
(ii) a circle if $\Delta \neq 0$, $h = 0$, $a = b$
(iii) a parabola if $\Delta \neq 0$, $ab - h^2 = 0$
(iv) an ellipse if $\Delta \neq 0$, $ab - h^2 > 0$
(v) a hyperbola if $\Delta \neq 0$, $ab - h^2 < 0$

STANDARD EQUATION OF A PARABOLA



- Let P be the moving point and A be the origin. Here SZ is the axis of the parabola.
  Now, the middle point of SZ, say A, will lie on the locus of P *i.e.*, $AS = AZ$.

Let $AS = a$, then $ZA = a$. Let (x, y) be the coordinates of point P.

$\therefore$ $MP = ZN = ZA + AN = a + x$. Also, $MP = PS$

$\Rightarrow$ $MP^2 = PS^2 \Rightarrow (a + x)^2 = (x - a)^2 + y^2$.

Hence, the equation of parabola is $y^2 = 4ax$.

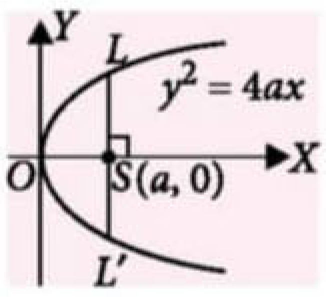
Double Ordinate and Latus Rectum of the Parabola

- Any chord of the parabola $y^2 = 4ax$ which is $\perp$ to its axis is called the double ordinate.

---

*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

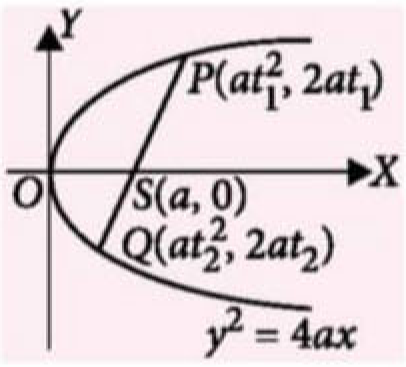
- The given parabola be $y^2 = 4ax$.
  In the figure LSL' is the latus rectum,
  Also, $LSL' = 2(\sqrt{4a \cdot a}) = 4a$
  = length of latus rectum or length of double ordinate through the focus S.


Y
L
$y^2 = 4ax$
O
S(a, 0)
X
L'

Focal Chord

Any chord to the parabola $y^2 = 4ax$ which passes through the focus is called a focal chord of the parabola $y^2 = 4ax$.

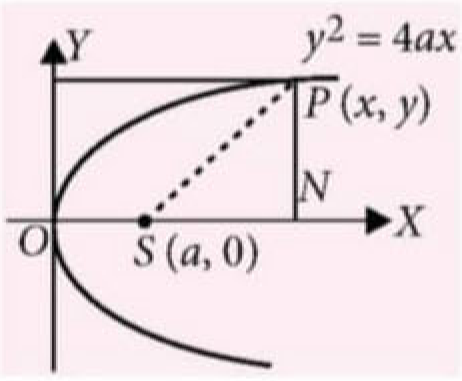
If the equation of chord PQ passes through $(a, 0)$, then $t_1t_2 = -1$.


Y
$P(at_1^2, 2at_1)$
O
S(a, 0)
X
$Q(at_2^2, 2at_2)$
$y^2 = 4ax$

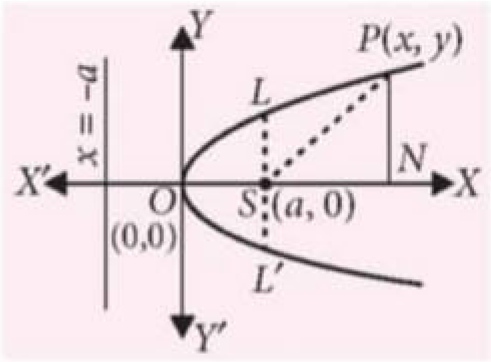
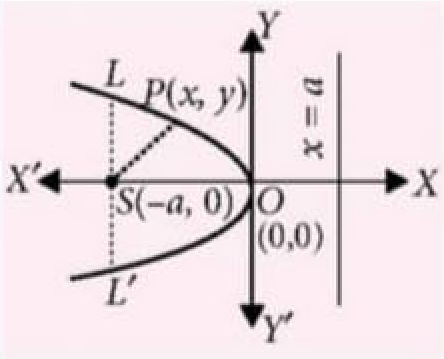
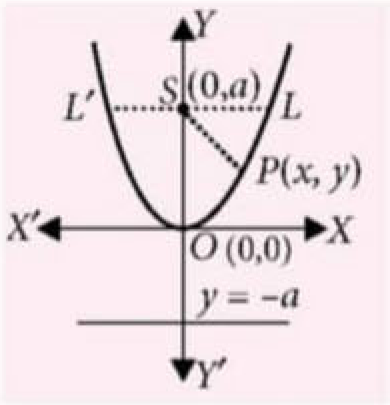
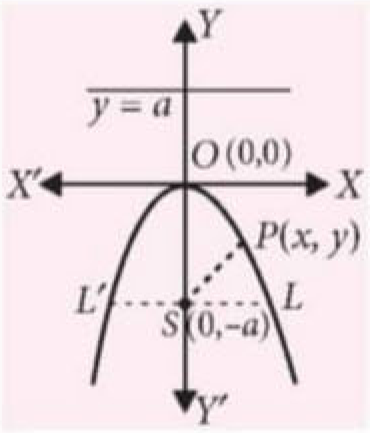
Focal Distance of Any Point

The distance between the point P on the conic and the focus S is called focal distance *i.e.*, PS.

$\therefore$ Focal distance $= PS = a + x$


Y
$y^2 = 4ax$
P (x, y)
N
O
S (a, 0)
X

Different Forms of The Parabola

Form	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Shape of the parabola	 Y P(x, y) x = −a L N X' O (0,0) S(a, 0) X L' Y'	 Y L P(x, y) x = a X' S(−a, 0) O (0,0) X L' Y'	 Y L' S(0,a) L P(x, y) X' O (0,0) X y = −a Y'	 Y y = a O (0,0) X' X P(x, y) L' S(0,−a) L Y'
Vertex	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation of directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Tangent at the vertex	$x = 0$	$x = 0$	$y = 0$	$y = 0$
Parametric Equation	$x = at^2, y = 2at$	$x = -at^2, y = 2at$	$x = 2at, y = at^2$	$x = 2at, y = -at^2$
Focal Distance of (x, y)	$x + a$	$-x + a$	$y + a$	$-y + a$

Equation of Tangent

At the point (x_1, y_1)	$yy_1 = 2a(x + x_1)$	$yy_1 = -2a(x + x_1)$	$xx_1 = 2a(y + y_1)$	$xx_1 = -2a(y + y_1)$
In terms of m	$y = mx + a/m$	$y = mx - a/m$	$y = mx - am^2$	$y = mx + am^2$
In terms of t	$yt = x + at^2$	$yt = -x + at^2$	$xt = y + at^2$	$xt = -y + at^2$

Equation of Normal

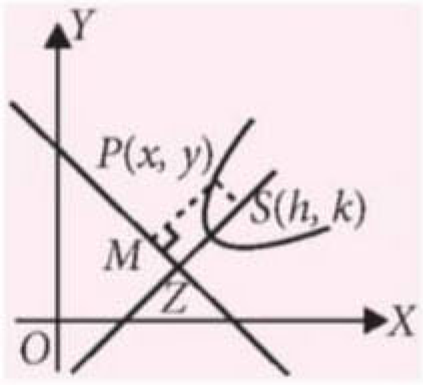
At the point (x_1, y_1)	$y - y_1 = \frac{-y_1}{2a}(x - x_1)$	$y - y_1 = \frac{y_1}{2a}(x - x_1)$	$x - x_1 = \frac{-x_1}{2a}(y - y_1)$	$x - x_1 = \frac{x_1}{2a}(y - y_1)$
In terms of m	$y = mx - 2am - am^3$	$y = mx + 2am + am^3$	$y = mx + 2a + \frac{a}{m^2}$	$y = mx - 2a - \frac{a}{m^2}$
In terms of t	$y = -tx + 2at + at^3$	$y = tx + 2at + at^3$	$x = -ty + 2at + at^3$	$x = ty + 2at + at^3$

Position of a Point Relative to the Parabola

Consider the parabola $y^2 = 4ax$ and $P(x_1, y_1)$ be a point.
If $y_1^2 - 4ax_1 = 0$, then the point P lies on the parabola.
If $y_1^2 - 4ax_1 > 0$, then P lies outside the parabola.
If $y_1^2 - 4ax_1 < 0$, then P lies inside the parabola.

GENERAL EQUATION OF A PARABOLA

Let (h, k) be the coordinates of focus S and $lx + my + n = 0$ be the equation of the directrix ZM of a parabola. Let (x, y) be the coordinates of any point P on the parabola.



Then the relation,
PS = distance of P from ZM, gives

$$(x - h)^2 + (y - k)^2 = (lx + my + n)^2/(l^2 + m^2)$$

$$\Rightarrow \quad (mx - ly)^2 + 2gx + 2fy + d = 0$$

which is the general equation of a parabola.

INTERSECTION OF A STRAIGHT LINE WITH THE PARABOLA

Points of intersection of parabola $y^2 = 4ax$ and line $y = mx + c$ are given by $(mx + c)^2 = 4ax$

i.e., $m^2x^2 + 2x(mc - 2a) + c^2 = 0$ ...(i)

Since (i) is a quadratic equation, the straight line meets the parabola in two points (real, coincident, or imaginary). The roots of (i) are real or imaginary according as $\{2(mc - 2a)\}^2 - 4m^2c^2$ is positive or negative, *i.e.* according as $-amc + a^2$ is positive or negative, *i.e.* according as mc is less than or greater than a.

- When $m = 0$, equation (i) gives $x = \dfrac{c^2}{4a}$ *i.e.*, only one value of x. So, line parallel to the axis of the parabola meets the parabola only in one point.

LENGTH OF THE CHORD

The abscissae of the points common to the straight line $y = mx + c$ and the parabola $y^2 = 4ax$ are given by the equation $m^2x^2 + (2mc - 4a)\,x + c^2 = 0$.

If (x_1, y_1) and (x_2, y_2) are the points of intersection, then

$$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1\,x_2 = \frac{16a(a - mc)}{m^4}$$

and $(y_1 - y_2) = m(x_1 - x_2)$

Hence, the required length $= \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$

$$= \sqrt{1 + m^2}\;|(x_1 - x_2)| = \frac{4}{m^2}\sqrt{1 + m^2}\;\sqrt{a(a - mc)}$$

EQUATION OF THE TANGENTS FROM AN EXTERNAL POINT

Let $y^2 = 4ax$ be the parabola and $P(x_1, y_1)$ be an external point. Then, equations of the tangents from (x_1, y_1) to the given parabola are given by

$SS_1 = T^2$, where $S = y^2 - 4ax$, $S_1 = y_1^2 - 4ax_1$, $T = yy_1 - 2a(x + x_1)$

Note : (1) The point of intersection of the tangents at 't_1' and 't_2' to the parabola $y^2 = 4ax$ is $(at_1t_2, a(t_1 + t_2))$.

(2) For the parabola, $y^2 = 4ax$ the point of contact of the tangent $y = mx + c$, where $c = \dfrac{a}{m}$ is given by $\left(\dfrac{a}{m^2}, \dfrac{2a}{m}\right)$.

CHORD OF CONTACT

Equation of the chord of contact of the tangents drawn from a point (x_1, y_1) to the parabola $y^2 = 4ax$ is $T = 0$, *i.e.* $yy_1 - 2a(x + x_1) = 0$.

Equation of a Chord with Midpoint (x_1, y_1)

The equation of the chord of the parabola $y^2 = 4ax$ with mid point (x_1, y_1) is $T = S_1$
i.e., $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$

Co-Normal Points

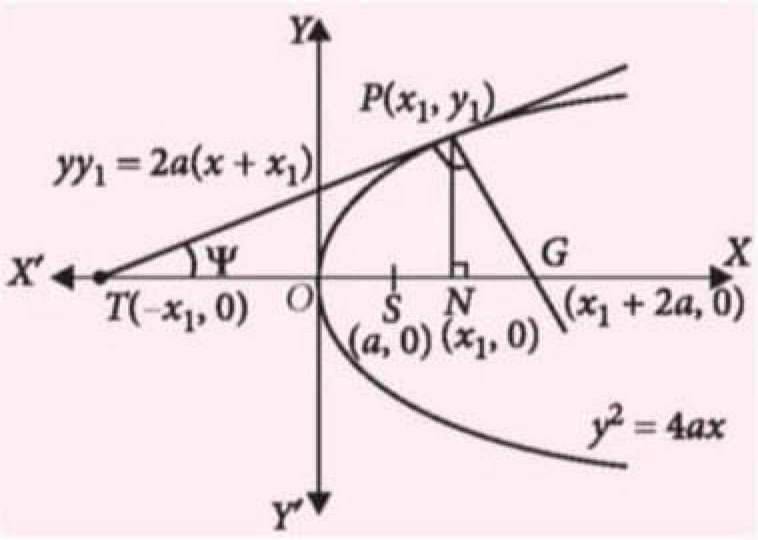
For the parabola $y^2 = 4ax$ and a point $P(x_1, y_1)$equation of normal is given by $y = mx - 2am - am^3$.
or $am^3 + m(2a - x_1) + y_1 = 0$, which is cubic in m and has three roots say m_1, m_2, m_3.

(i) $m_1 + m_2 + m_3 = 0$
(ii) $m_1\,m_2 + m_2\,m_3 + m_3\,m_1 = (2a - x_1)/a$
(iii) $m_1\,m_2\,m_3 = -\,y_1/a$

Note: (1) If normal at the point t_1 meets the parabola again at t_2, then $t_2 = -t_1 - 2/t_1$.
(2) Point of intersection of the normals to the parabola $y^2 = 4ax$ at $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ is $(2a + a(t_1^2 + t_2^2 + t_1t_2), -\,at_1t_2(t_1 + t_2))$.

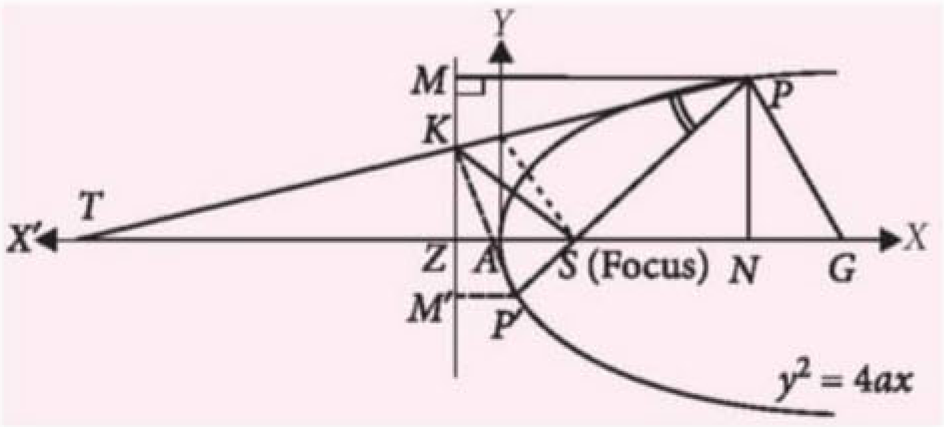
Subtangent and Subnormal

Let the tangent and normal at any point $P(x_1, y_1)$ on the parabola $y^2 = 4ax$ meet the axis in T and G respectively. Then PT is called the length of the tangent at P and PG is called the length of the normal at P.



NT is subtangent and NG is subnormal at P. The coordinates of T and G can be easily found by putting $y = 0$ in the equations of the tangent and normal at P. It is evident from figure that length of
Subtangent = NT = twice the abscissa of point P
Subnormal = $NG = 2a$ = length of semi latus rectum

PROPERTIES OF THE PARABOLA

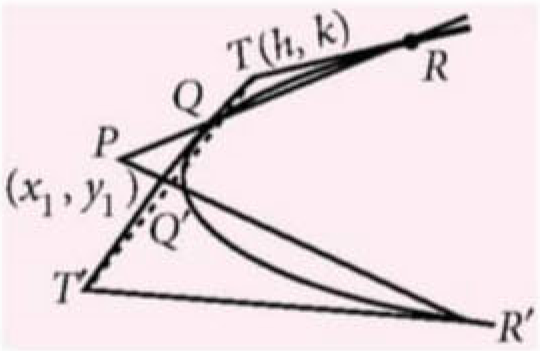


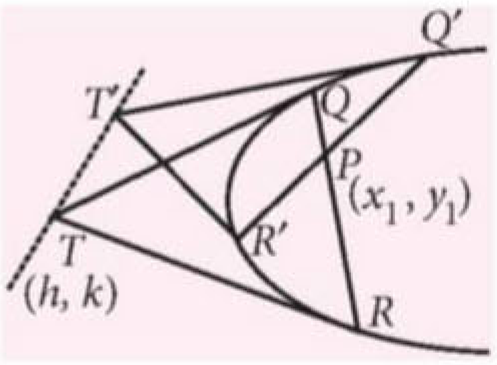
- The tangent at any point P of a parabola bisects the angle between the focal chord through P and the perpendicular from P on the directrix.
- The portion of a tangent to a parabola cut off between the directrix and the curve subtends a right angle at the focus.
- Tangents at the extremities of a focal chord intersect at right angles on the directrix.
- Any tangent to a parabola and the perpendicular on it from the focus meet on the tangent at the vertex.

Pole and Polar

Let Q and R be the points on which any chord drawn through the point P, whose coordinates are (x_1, y_1), meets the parabola.

Let the tangents at Q and R meet on the point whose coordinates are (h, k).





We require the locus of (h, k).

Since QR is the chord of contact of tangents from (h, k), so its equation is $ky = 2a\,(x + h)$

Since this straight line passes through the point (x_1, y_1). $\therefore\ \ ky_1 = 2a\,(x_1 + h)$

It follows that the point (h, k) always lies on the straight line $yy_1 = 2a(x + x_1)$.

The locus of the point of intersection of the tangents to the parabola at the ends of a chord drawn from a fixed point P is called the polar of point P and the point P is called the pole of the polar.

PROBLEMS

Single Correct Answer Type

1. If (0, 4) and (0, 2) are respectively the vertex and focus of a parabola, then its equation is

(a) $x^2 + 8y = 32$ (b) $y^2 + 8x = 32$
(c) $x^2 - 8y = 32$ (d) $y^2 - 8x = 32$

2. Co-ordinates of the focus of the parabola $x^2 - 4x - 8y - 4 = 0$ are

(a) (0, 2) (b) (2, 1) (c) (1, 2) (d) (–2, –1)

3. If $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ are two variable points on the curve $y^2 = 4ax$ and PQ subtends a right angle at the vertex, then t_1t_2 is equal to

(a) –1 (b) –2 (c) –3 (d) –4

4. In the parabola $y^2 = 4ax$, the length of the chord passing through the vertex and inclined to the axis at an angle $\pi/4$ is

(a) $4a\sqrt{2}$ (b) $\dfrac{4a}{\sqrt{2}}$
(c) $2a\sqrt{2}$ (d) none of these

5. The coordinates of a point on the parabola $y^2 = 8x$, whose focal distance is 4, are

(a) $\left(\dfrac{1}{2}, \pm 2\right)$ (b) $(1, \pm 2\sqrt{2})$
(c) $(2, \pm 4)$ (d) none of these

6. Two tangents are drawn from the point (–2, –1) to the parabola $y^2 = 4x$. If α is the angle between these tangents, then $\tan\alpha$ equals to

(a) 3 (b) 1/3 (c) 2 (d) 1/2

7. The angle between tangents to the parabola $y^2 = 4ax$ at the point where it intersects with the line $x - y - a = 0$ is

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/2$

8. The focus of the parabola $x^2 - 8x + 2y + 7 = 0$ is

(a) $\left(0, -\dfrac{1}{2}\right)$ (b) $\left(4, \dfrac{9}{2}\right)$
(c) $(4, 4)$ (d) $\left(-4, -\dfrac{9}{2}\right)$

9. For the parabola $y^2 = 4ax$, the ratio of length of the subtangent to the abscissa is

(a) 1 : 1 (b) 2 : 1 (c) $x : y$ (d) $x^2 : y^2$

10. The length of subnormal to the parabola $y^2 = 4ax$ at any point is equal to

(a) $a\sqrt{2}$ (b) $2\sqrt{2a}$ (c) $a/\sqrt{2}$ (d) $2a$

11. The length of the latus rectum of the parabola $x = ay^2 + by + c$ is

(a) $a/4$ (b) $a/3$ (c) $1/a$ (d) $1/4a$

12. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum is

(a) 12 sq. units (b) 16 sq. units
(c) 18 sq. units (d) 24 sq. units

13. If the line $x + y - 1 = 0$ touches the parabola $y^2 = kx$, then the value of k is

(a) 4 (b) -4 (c) 2 (d) -2

14. The coordinates of the point on the parabola $y = x^2 + 7x + 2$, which is nearest to the straight line $y = 3x - 3$ are

(a) $(-2, -8)$ (b) $(1, 10)$ (c) $(2, 20)$ (d) $(-1, -4)$

15. The point (1, 2) is one extremity of focal chord of parabola $y^2 = 4x$. The length of this focal chord is

(a) 2 (b) 4

(c) 6 (d) none of these

16. If the straight line $y = mx + c$ $(m > 0)$ touches the parabola $y^2 = 8(x + 2)$, then the minimum value taken by c is

(a) 12 (b) 8 (c) 4 (d) 6

17. The length of the subtangent to the parabola $y^2 = 16x$ at the point whose abscissa is 4, is

(a) 2 (b) 4

(c) 8 (d) none of these

18. If the line $x + y = 1$ touches the parabola $y^2 - y + x = 0$, then the co-ordinates of the point of contact are

(a) $(1, 1)$ (b) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (c) $(0, 1)$ (d) $(1, 0)$

19. If (x_1, y_1) and (x_2, y_2) are the ends of a focal chord of the parabola $y^2 = 4ax$, then $x_1x_2 + y_1y_2 =$

(a) $-3a^2$ (b) $3a^2$ (c) $-4a^2$ (d) $4a^2$

20. If the segment intercepted by the parabola $y^2 = 4ax$ with the line $lx + my + n = 0$ subtends a right angle at the vertex, then

(a) $4am + n = 0$ (b) $4am + 4al + n = 0$

(c) $4al + n = 0$ (d) $al + n = 0$

21. Equation of parabola having its vertex at $A(1,0)$ and focus at $S(3, 0)$ is

(a) $y^2 = 4(x - 1)$ (b) $y^2 = 4(x + 1)$

(c) $y^2 = 8(x - 1)$ (d) $y^2 = 8(x + 1)$

22. Equation of parabola having the extremities of its latus rectum as (3, 4) and (4, 3) is

(a) $\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{(x + y - 5)^2}{2}$

(b) $\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{(x + y - 8)^2}{2}$

(c) $\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{(x + y - 4)^2}{2}$

(d) None of these

23. Locus of mid-point of chords of the parabola $y^2 = 4ax$ that pass through the point $(3a, a)$ is

(a) $y^2 + 2ax + ay - 6a^2 = 0$

(b) $y^2 + 2ax - ay + 6a^2 = 0$

(c) $y^2 - 2ax + ay + 6a^2 = 0$

(d) $y^2 - 2ax - ay + 6a^2 = 0$

24. If the line joining the points $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$ passes through $C(0, b)$, then

(a) $b(t_1 + t_2) = 2at_1t_2$ (b) $2b(t_1 + t_2) = at_1t_2$

(c) $b(t_1 + t_2) = at_1t_2$ (d) None of these

25. If the lines $(y - b) = m_1(x + a)$ and $(y - b) = m_2(x + a)$ are the tangents of $y^2 = 4ax$, then

(a) $m_1 + m_2 = 0$ (b) $m_1m_2 = 1$

(c) $m_1m_2 = -1$ (d) $m_1 + m_2 = 1$

26. Tangents drawn to parabola $y^2 = 4ax$ at the points A and B intersect at C. Ordinate of A, C and B are in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

27. Tangents drawn to parabola $y^2 = 4ax$ at the point A and B intersect at C. If 'S' be the focus of the parabola then, SA, SC and SB are in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

28. Locus of trisection point of any arbitrary double ordinate on the parabola $x^2 = 4by$ is

(a) $9x^2 = by$ (b) $3x^2 = 2by$

(c) $9x^2 = 4by$ (d) $9x^2 = 2by$

29. If three distinct and real normals can be drawn to $y^2 = 8x$ from the point $(a, 0)$, then

(a) $a > 2$ (b) $a > 4$

(c) $a \in (2, 4)$ (d) None of these

30. The parabola $y^2 = 4x$ and the circle $(x - 6)^2 + y^2 = r^2$ will have no common tangent if 'r' is equal to

(a) $r > \sqrt{20}$ (b) $r < \sqrt{20}$

(c) $r > \sqrt{18}$ (d) $r \in (\sqrt{20}, \sqrt{28})$

31. Parabola $y^2 = 4x$ and the circle having its centre at (6, 5) intersect at right angle. Possible point of intersection of these curves can be

(a) $(9, 6)$ (b) $(2, \sqrt{8})$

(c) $(1, 2)$ (d) $(3, 2\sqrt{3})$

32. Normal PO, PA and PB ('O' being the origin) are drawn to $y^2 = 4x$ from $P(h, 0)$. If $\angle AOB = \pi/2$, then area of quadrilateral $OAPB$ is equal to

(a) 12 sq. units (b) 24 sq. units

(c) 6 sq. units (d) 18 sq. units

33. Radius of the circle that passes through origin and touches the parabola $y^2 = 4ax$ at the point $(a, 2a)$ is

(a) $\frac{5}{\sqrt{2}}a$ (b) $2\sqrt{2}a$ (c) $5\sqrt{2}a$ (d) $3\sqrt{2}a$

34. If the line $y = mx + c$ is a tangent to $y^2 = 4mx$, then distance of this tangent from the parallel normal is

(a) $\sqrt{1+m^2}$ (b) $\sqrt{2+m^2}$

(c) $(1 + m^2)^{3/2}$ (d) $(2 + m^2)^{3/2}$

35. Angle between the tangents drawn to $y^2 = 4x$ at the points where it is intersected by the line $y = x - 1$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

36. Maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ can be equal to

(a) 3 (b) 4 (c) 6 (d) 5

37. If two different tangents of $y^2 = 4x$ are the normals to $x^2 = 4by$, then

(a) $|b| > \frac{1}{2\sqrt{2}}$ (b) $|b| < \frac{1}{2\sqrt{2}}$

(c) $|b| > \frac{1}{\sqrt{2}}$ (d) $|b| < \frac{1}{\sqrt{2}}$

Multiple Correct Answer Type

38. A circle of radius 'r' touches the parabola $y^2 + 4x = 0$ at the vertex of the parabola. The centre of the circle lies to the left of the vertex and this circle lies completely within the parabola then exhaustive range of 'r' belongs to

(a) $\left(1, \frac{3}{2}\right)$ (b) $(0, 2)$ (c) $\left(0, \frac{5}{2}\right)$ (d) $(0, 3)$

39. If $f(x + y) = f(x).f(y)$ for all x, y and $f(1) = 2$ and $a_r = f(r)$, for $r \in N$, then the coordinates of a point on the parabola $y^2 = 8x$ whose focal distance is 4, may be

(a) (a_1, a_2) (b) $(a_1, -a_2)$ (c) (a_1, a_1) (d) (a_2, a_2)

40. If the parabolas $y^2 = 4kx (k > 0)$ and $y^2 = 4(x - 1)$ do not have a common normal other than the axis of parabola, then $k \in$

(a) $(0, 1)$ (b) $(2, \infty)$ (c) $(3, \infty)$ (d) $(0, \infty)$

41. The points on axis of parabola $3x^2 + 4x - 6y + 8 = 0$ from which three distinct normals can be drawn to it are

(a) $\left(\frac{-2}{3}, 2\right)$ (b) $\left(\frac{-2}{3}, 3\right)$

(c) $\left(\frac{-2}{3}, 4\right)$ (d) $\left(\frac{-2}{3}, 1\right)$

Assertion & Reason Type

Directions : *In the following questions, Statement-1 is followed by Statement-2. Mark the correct choice as :*

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement–2 is true, Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

42. Statement-1 : The normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in (h, k), then centroid of ΔPQR lies on $x = 0$.

Statement-2 : If A, B, C are three co-normal points of the parabola $y^2 = 4ax$, then centroid of ΔABC lies on axis of the parabola.

43. Statement-1 : Let 'a' and 'b' be the segments of any focal chord and $2l$ be its latus rectum, then $a^3 + b^3 > 2l^3$.

Statement-2 : For distinct positive numbers a and b A.M. > G.M. > H.M.

44. Statement-1 : The normals at the points where $t = \pm\sqrt{2}$, on the parabola $y^2 = 4ax$ subtends a right angle at the vertex of the parabola.

Statement-2 : For a normal chord of the parabola $y^2 = 4ax$ having ends t_1, t_2 we always have $t_1.t_2 = 2$.

Comprehension Type

Paragraph for Q. No. 45 to 47

If a source of light is placed at the fixed point of a parabola and if the parabola is a reflecting surface, then the ray will bounce back in a line parallel to the axis of the parabola.

45. A ray of light is coming along the line $y = 2$ from the positive direction of x-axis and strikes a concave mirror whose intersection with the xy-plane is a parabola $y^2 = 8x$, then the equation of the reflected ray is

(a) $2x + 5y = 4$ (b) $3x + 2y = 6$

(c) $4x + 3y = 8$ (d) $5x + 4y = 10$

46. A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $y^2 + 10y - 4x + 17 = 0$. After reflection, the ray must pass through the point

(a) $(-2, -5)$ (b) $(-1, -5)$ (c) $(-3, -5)$ (d) $(-4, -5)$

47. Two rays of light coming along the lines $y = 1$ and $y = -2$ form the positive direction of x-axis and strikes a concave mirror whose intersection with the xy-plane is a parabola $y^2 = x$ at A and B respectively. The reflected rays pass through a fixed point C, then the area of the triangle ABC is

(a) $\frac{21}{8}$ sq. unit (b) $\frac{19}{2}$ sq. unit

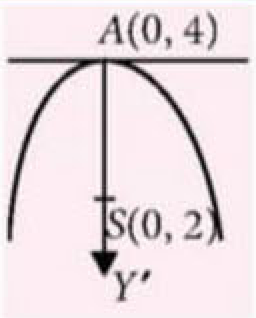
(c) $\frac{17}{2}$ sq. unit (d) $\frac{15}{2}$ sq. unit

Matrix-Match Type

48. Match the following :

Column-I		Column-II	
A.	If two distinct chords of a parabola $y^2 = 4ax$ passing through the point $(a, 2a)$ are bisected by the line $x + y = 1$, then the length of the latus rectum can be	(p)	-1
B.	The parabola $y = x^2 - 5x + 4$ cuts the x-axis at P and Q. A circle is drawn through P and Q so that the origin lies outside it. The length of tangent to the circle from the origin is equal to	(q)	0
C.	If $y + b = m_1(x + a)$ and $y + b = m_2(x + a)$ are two tangents to $y^2 = 4ax$, then m_1m_2 is equal to	(r)	1
D.	If the point $(h, -1)$ is exterior to both the parabolas $y^2 = \lvert x \rvert$, then the integral part of h can be equal to	(s)	2

SOLUTIONS



1. (a) : $AS = 2 = a$.

Vertex (0, 4) lies on y-axis.

As focus is below the vertex, so it is a downward parabola of the form $(x - h)^2 = -4a(y - k)$.

$\Rightarrow \quad (x - 0)^2 = -4 \times 2(y - 4) \quad \Rightarrow \quad x^2 + 8y = 32$

2. (b) : $(x - 2)^2 = 8(y + 1)$

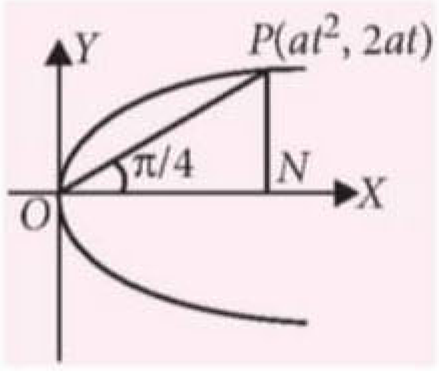
Focus: $x - 2 = 0 \Rightarrow x = 2$

Similarly, $y + 1 = 2 \Rightarrow y = 1$

$\therefore$ Focus is (2, 1)

3. (d) : Slope of $OP = \frac{2}{t_1}$ and Slope of $OQ = \frac{2}{t_2}$

Given $OP \perp OQ \Rightarrow \frac{2}{t_1} \cdot \frac{2}{t_2} = -1 \Rightarrow t_1t_2 = -4$



4. (a) : Since $\frac{2at}{at^2} = \tan\frac{\pi}{4}$

$\Rightarrow \quad t = 2 \quad \Rightarrow \quad P = (4a, 4a)$

$\therefore \quad OP = 4\sqrt{2}a$

5. (c) : Focal distance of a point $P(x, y)$ on $y^2 = 4ax$ is $(x + a)$

i.e., $4 = x + 2 \Rightarrow x = 2$

$\therefore \quad y^2 = 8 \times 2 = 16 \Rightarrow y = \pm 4$

6. (a) : Here $a = 1$. Any tangent is $y = mx + \frac{1}{m}$.

It passes through $(-2, -1)$ $\therefore \quad 2m^2 - m - 1 = 0$

$$\Rightarrow \quad m = 1, -\frac{1}{2} \Rightarrow \tan\alpha = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$$

7. (d) : The given chord is a focal chord as it passes through focus $(a, 0)$ and we know that tangents at the extremities of a focal chord meet at right angles on the directrix.

8. (c) : Given parabola is $(x - 4)^2 = -2\left(y - \frac{9}{2}\right)$

Focus is given by $x - 4 = 0$,

$y - \frac{9}{2} = -\frac{1}{2} \Rightarrow$ Focus is (4, 4).

9. (b) : $\frac{\text{Length of subtangent}}{x} = \frac{y}{y'x}$ and $2yy' = 4a$

$$\therefore \quad y' = \frac{2a}{y} \Rightarrow \frac{\text{Length of subtangent}}{x} = \frac{4ax}{2ax} = \frac{2}{1}$$

10. (d) : Length of subnormal at point (x_1, y_1)

$$= y_1\frac{dy}{dx} = y_1\left(\frac{2a}{y_1}\right) = 2a.$$

11. (c) : $x = a\left(y^2 + \frac{b}{a}y + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c$

or $a\left(y + \frac{b}{2a}\right)^2 = x + \frac{b^2}{4a} - c$

$$\Rightarrow \quad \left(y + \frac{b}{2a}\right)^2 = \frac{1}{a}\left(x + \frac{b^2 - 4ac}{4a}\right)$$

$\therefore \quad$ Length of latus rectum $= \frac{1}{a}$

12. (c) : Required area, $\Delta = \frac{1}{2}(12 \times 3) = 18$ sq. units

13. (b) : $y = mx + k/4m$

Comparing it with the given line $y = 1 - x$,

we get $m = -1$ and $k/4m = 1 \Rightarrow k = -4$

14. (a) : Any point on the parabola is $(x, x^2 + 7x + 2)$

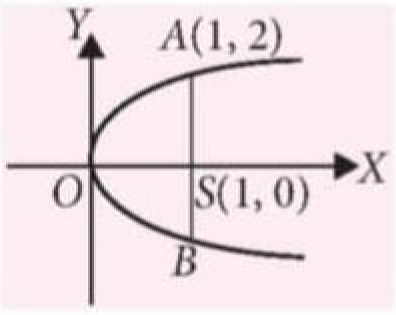
Its distance from the line $y = 3x - 3$ is given by

$$P = \left|\frac{3x - (x^2 + 7x + 2) - 3}{\sqrt{9 + 1}}\right| = \left|\frac{x^2 + 4x + 5}{\sqrt{10}}\right|$$

$= \frac{x^2 + 4x + 5}{\sqrt{10}}$ $(\because x^2 + 4x + 5 > 0 \text{ for all } x \in R)$

Now, $\frac{dP}{dx} = 0 \Rightarrow x = -2$ $\therefore$ Required point $\equiv (-2, -8)$

15. (b) : The parabola is $y^2 = 4x$.
Here, $a = 1$ and focus is $(1, 0)$.
The focal chord (ASB) is clearly latus rectum of parabola.
$\therefore$ Length of latus rectum = 4



16. (c) : The tangent of slope m must be of the form

$$y = m(x + 2) + \frac{a}{m}.$$

So, $2m + \frac{2}{m} = c \Rightarrow c = 2\left(m + \frac{1}{m}\right) \geq 2 \times 2$. So $c_{\min} = 4$.

17. (c) : The length of subtangent $= 2x_1 = 2 \times 4 = 8$.

18. (c) : The tangent at (x_1, y_1) is $2yy_1 - y - y_1 + x + x_1 = 0$.
Comparing it with $x + y - 1 = 0$, we get
$(x_1, y_1) \equiv (0, 1)$.

19. (a) : $x_1x_2 + y_1y_2 = a^2t_1^2t_2^2 + 4a^2t_1t_2 = a^2 - 4a^2 = -3a^2$.

20. (c) : $y^2 + 4ax\left(\frac{lx + my}{n}\right) = 0$

Now, coefficient of x^2 + coefficient of $y^2 = 0$
$\Rightarrow 4al + n = 0$.

21. (c) : Clearly, axis of parabola is the x-axis.
Corresponding value of $a = 3 - 1 = 2$.
Thus, equation of parabola is $y^2 = 8(x - 1)$.

22. (b) : Focus is $\left(\frac{7}{2}, \frac{7}{2}\right)$ and its axis is the line $y = x$.

Corresponding value of 'a' $= \frac{1}{4}(\sqrt{1+1}) = \frac{\sqrt{2}}{4}$. Let the equation of its directrix be $y + x + \lambda = 0$.

$$\Rightarrow \frac{|3 + 4 + \lambda|}{\sqrt{2}} = 2 \cdot \frac{\sqrt{2}}{4} \quad \Rightarrow \quad \lambda = -6, -8$$

Thus equation of parabola is

$$\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{(x + y - 6)^2}{2}$$

$$\text{or} \quad \left(x - \frac{7}{2}\right)^2 + \left(x - \frac{7}{2}\right)^2 = \frac{(x + y - 8)^2}{2}$$

23. (d) : Let the mid point of chord be $P(h, k)$ then its equation is $T = S_1$
i.e., $yk - 2a(x + h) = k^2 - 4ah$
It must pass through $(3a, a)$.
$\therefore$ $ak - 2a(3a + h) = k^2 - 4ah$
Thus, locus of P is $y^2 - 2ax - ay + 6a^2 = 0$

24. (a) : Equation of line joining A, B is
$y(t_1 + t_2) = 2(x + at_1t_2)$
It will pass through $(0, b)$, if

$$b(t_1 + t_2) = 2at_1t_2 \quad \Rightarrow \quad t_1 + t_2 - \frac{2a}{b}t_tt_2 = 0$$

25. (c) : Clearly, both the lines pass through $(-a, b)$ which is a point lying on the directrix of the parabola. Thus, $m_1m_2 = -1$.

26. (a) : If $A \equiv (at_1^2, 2at_1)$, $B \equiv (at_2^2, 2at_2)$, then
$C \equiv (at_1t_2, a(t_1 + t_2))$
Clearly the ordinates of A, C, B are in A.P.

27. (b) : If $A \equiv (at_1^2, 2at_1)$, $B \equiv (at_2^2, 2at_2)$, then
$C \equiv (at_1t_2, a(t_1 + t_2))$
Now, $SA = a + at_1^2$, $SB = a + at_2^2$,

$$SC = \sqrt{(at_1t_2 - a)^2 + a^2(t_1 + t_2)^2}$$

$$= a\sqrt{(t_1t_2 - 1)^2 + (t_1 + t_2)^2}$$

$$= a\sqrt{t_1^2 + t_2^2 + 1 + t_1^2t_2^2} = a\sqrt{(1 + t_1^2)(1 + t_2^2)}$$

Clearly, $SC^2 = SA \cdot SB$

28. (c) : Let $A \equiv (2bt, bt^2)$, $B \equiv (-2bt, bt^2)$ be the extremities of the double ordinate AB. If $C(h, k)$ be its trisection point, then $3h = 4bt - 2bt$, $3k = 2bt^2 + bt^2$.

$$\Rightarrow \quad t = \frac{3h}{2b}, t^2 = \frac{k}{b} \quad \Rightarrow \quad \frac{k}{b} = \frac{9h^2}{4b^2}$$

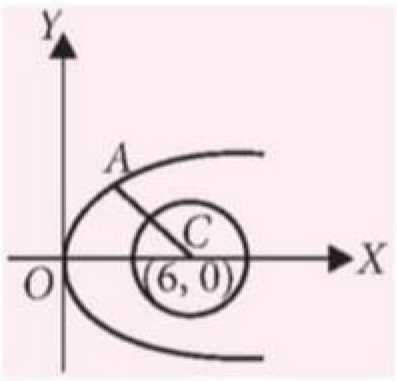
Thus locus of C is $9x^2 = 4by$

29. (b) : Equation of normal in terms of m is
$y = mx - 4m - 2m^3$
If it passes through $(a, 0)$, then
$am - 4m - 2m^3 = 0 \quad \Rightarrow \quad m(a - 4 - 2m^2) = 0$

$$\Rightarrow \quad m = 0, m^2 = \frac{a - 4}{2}.$$

For three distinct normals, $a - 4 > 0 \quad \Rightarrow \quad a > 4$.

30. (b) : Any normal of parabola is
$y = -tx + 2t + t^3$
If it pass through $(6, 0)$, then
$-6t + 2t + t^3 = 0$
$\Rightarrow \quad t = 0, t^2 = 4 \therefore A \equiv (4, 4)$.
Thus for no common tangent,
$AC = \sqrt{4 + 16} > r \quad \Rightarrow \quad r < \sqrt{20}$

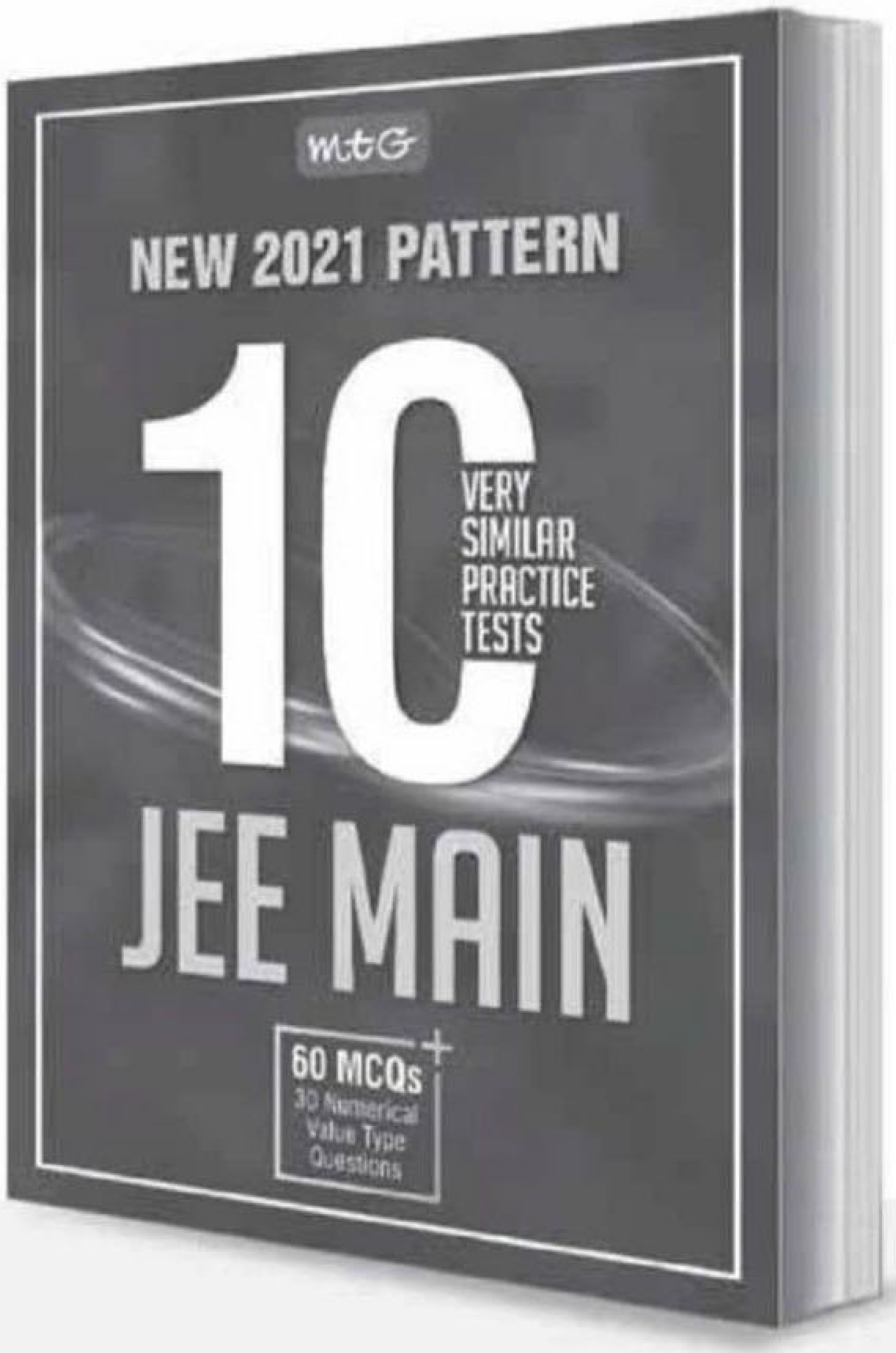


31. (a) : Let the possible point be $(t^2, 2t)$
Equation of tangent at this point is $yt = x + t^2$
It must pass through $(6, 5)$
$\therefore \quad t^2 - 5t + 6 = 0 \quad \Rightarrow \quad t = 2, 3$
Thus possible points are $(4, 4)$, $(9, 6)$.

mtG

10 Very Similar Practice Tests
JEE Main

Visit www.mtg.in for latest offers and to buy online!


mtG
NEW 2021 PATTERN
10
VERY SIMILAR PRACTICE TESTS
JEE MAIN
60 MCQs+
30 Numerical Value Type Questions

₹ 200/-

HIGHLIGHTS

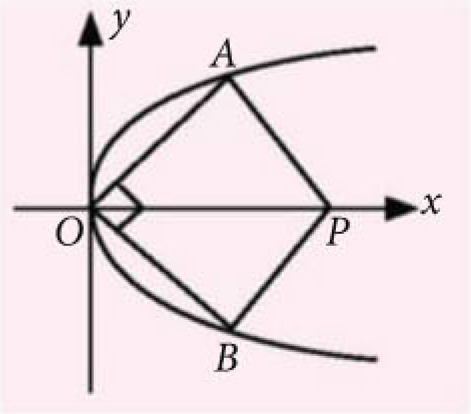
- 10 Very Similar Practice Tests as per the latest pattern of NTA JEE Main (60 MCQs+ 30 Numerical Value Type Questions)
- OMR sheet provided at the end of each test
- Detailed solutions of each practice test included

Available at all leading book shops throughout India. To buy online visit www.mtg.in.
For more information or for help in placing your order,
call 0124-6601200, 1800-10-38673 (Toll Free) or e-mail *info@mtg.in*

Also available on Flipkart, Snapdeal, Amazon, Infibeam on cash on delivery

32. (b) : Let $A \equiv (t^2, 2t)$, $B \equiv (t^2, -2t)$

$$m_{OA} = \frac{2}{t},\quad m_{OB} = -\frac{2}{t}$$



$\Rightarrow\ t^2 = 4$

Equation of normal AP is

$$y = -2x + 4 + 8$$

$\Rightarrow\ P \equiv (6, 0)$

$\therefore$ ar. $(OAPB) = \frac{1}{2}(OP)(AB) = \frac{1}{2}\cdot 6\cdot 8 = 24$ sq. units.

33. (a) : Equation of tangent of parabola at $(a, 2a)$ is

$$y\cdot 2a = 2a(x + a),\ i.e.,\ y - x - a = 0.$$

Equation of circle touching the parabola at $(a, 2a)$ is $(x - a)^2 + (y - 2a)^2 + \lambda\,(y - x - a) = 0$

Since, it passes through (0, 0), therefore

$$a^2 + 4a^2 + \lambda\,(-a) = 0 \quad\Rightarrow\quad \lambda = 5a.$$

Thus required circle is $x^2 + y^2 - 7ax + ay = 0$

Its radius $= \sqrt{\frac{49}{4}a^2 + \frac{a^2}{4}} = \frac{5}{\sqrt{2}}a.$

34. (c) : Equation of parallel normal is

$$y = mx - 2m^2 - m^4$$

$\Rightarrow$ Required distance $= \dfrac{|c + 2m^2 + m^4|}{\sqrt{1+m^2}}$

Here, $c = 1$

Thus required distance $= (1 + m^2)^{3/2}$

35. (d) : The line $y = x - 1$ passes through (1, 0), that means it is a focal chord. Hence the required angle is $\pi/2$.

36. (d) : Normals to $y^2 = 4ax$ and $x^2 = 4by$ in terms of 'm' are $y = mx - 2am - am^3$ and $y = mx + 2b + \frac{b}{m^2}$.

For a common normal

$$2b + \frac{b}{m^2} + 2am + am^3 = 0$$

$\Rightarrow\ am^5 + 2am^3 + 2bm^2 + b = 0$

$\therefore$ There can be atmost 5 common normals.

37. (b) : Tangent to $y^2 = 4x$ in terms of m is

$$y = mx + \frac{1}{m}.$$

Normal to $x^2 = 4by$ in terms of m is

$$y = mx + 2b + \frac{b}{m^2}.$$

If these are same lines, then

$$\frac{1}{m} = 2b + \frac{b}{m^2}$$

$\Rightarrow\ 2bm^2 - m + b = 0$

For two different tangents, we must have

$$1 - 8b^2 > 0 \quad\Rightarrow\quad |b| < \frac{1}{\sqrt{8}}$$

38. (a, b) : Equation of circle is $(x + r)^2 + y^2 = r^2$

Solving it with $y^2 + 4x = 0$, we get $x = 0$ and $x = 4 - 2r$

The circle lies completely inside the parabola, $4 - 2r$ is not less than zero $4 - 2r \ge 0 \Rightarrow r \le 2$

39. (a, b) : Given parabola is $y^2 = 8x$ ...(i)

Here, $a = 2$. Let $P(2t^2, 4t)$ be a point on parabola

Given, $SP = 4$

$\therefore\ a(1 + t^2) = 4 \quad\Rightarrow\quad 2(1 + t^2) = 4 \quad\Rightarrow\quad t = \pm 1$

$\therefore\ P \equiv (2, 4)$ or $(2, -4)$

Given, $f(x + y) = f(x)f(y)$ for all x and y

Also, we have $f(1) = 2$ ...(ii)

From (i) and (ii), $f(2) = f(1 + 1) = f(1)\cdot f(1) = 2^2 = 4$

Similarly, $f(n) = 2^n$

Hence, $P \equiv (a_1, a_2)$ or $(a_1, -a_2)$.

40. (a, b, c) : If the parabolas have a common normal of slope 'm' (the only allowed value of m is $m = 0$) then it is given by $y = mx - 2km - km^3$

and $y = m\,(x - 1) - 2m - m^3 = mx - 3m - m^3$

$\Rightarrow\ 2km + km^3 = 3m + m^3 \Rightarrow m = 0, m^2 = \dfrac{3-2k}{k-1}.$

If $m^2 < 0$ then the only common normal is the axis.

$\Rightarrow\ \dfrac{3-2k}{k-1} < 0 \Rightarrow (k-1)(2k-3) > 0 \Rightarrow k > \dfrac{3}{2}$

or $k < 1$ and $k > 0$

41. (b, c) : The parabola can be written as

$$\left(x+\frac{2}{3}\right)^2 = 2\left(y-\frac{10}{9}\right) i.e.,\ X^2 = 2Y\left(X = x+\frac{2}{3}, Y = y-\frac{10}{9}\right).$$

A point on axis is $\left(\frac{-2}{3}, Y\right)$ from which three normals can be drawn if $Y > 1 \Rightarrow\ y > \frac{19}{9}$.

42. (d) : Let $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$, $R(at_3^2, 2at_3)$ be the three points on parabola.

Equation of the normal at 't' is $y + xt = 2at + at^3$

The above normal is passing through (h, k)

$\Rightarrow\ k + ht = 2at + at^3 \quad\Rightarrow\quad at^3 + (2a - h)t - k = 0$

The above is a cubic equation in 't' whose roots are $t_1, t_2, t_3 \Rightarrow \Sigma t_1 = 0$

Centroid of ΔPQR is $\left(\dfrac{a(t_1^2 + t_2^2 + t_3^2)}{3}, \dfrac{2a}{3}(t_1 + t_2 + t_3)\right)$

$$= \left(\frac{a(t_1^2 + t_2^2 + t_3^2)}{3}, 0\right)$$

Centroid lies on $y = 0$
Hence Statement - 1 is false but Statement - 2 is true.

43. (b) : Since semi latus rectum is the H.M. between focal segments

$\Rightarrow$ 'l' is H.M. of a, b

Now, G.M. >H.M.

$$\sqrt{ab} > \frac{2ab}{a+b} \Rightarrow \sqrt{ab} > l \Rightarrow ab > l^2$$

Now, A.M. > G.M.

$$\frac{a^3+b^3}{2} > \sqrt{a^3b^3} > l^3 \Rightarrow a^3+b^3 > 2l^3$$

44. (c) : When $P(t_1)$, $Q(t_2)$ are the ends of a normal chord then the product of slopes of AP and AQ, where A is the vertex, is

$$\frac{2}{t_1}\cdot\frac{2}{t_2} = \frac{4}{t_1\left(-t_1-\frac{2}{t_1}\right)} = \frac{-4}{t_1^2+2} = -1, \text{ when } t_1 = \pm\sqrt{2}$$

$\therefore$ Statement-1 is true but statement-2 is false.

45. (c) : Point of intersection of $y = 2$ and $y^2 = 8x$ is $P\left(\frac{1}{2}, 2\right)$ and focus of the parabola is $S(2, 0)$

$\therefore$ Equation of the reflected ray is

$$y - 0 = \frac{2-0}{1/2-2}(x-2)$$

$$\Rightarrow \quad y = -\frac{4}{3}(x-2) \quad \Rightarrow \quad 4x + 3y = 8$$

46. (b) : $\because$ $y^2 + 10y - 4x + 17 = 0$

$\Rightarrow$ $(y + 5)^2 - 25 - 4x + 17 = 0$

$\Rightarrow$ $(y + 5)^2 = 4x + 8 \Rightarrow (y + 5)^2 = 4(x + 2)$

Let $y + 5 = Y$, $x + 2 = X$, then $Y^2 = 4X$

Focus is $X = 1$, $Y = 0$, *i.e.*, $(-1, -5)$

After reflection, the ray must pass through focus $(-1, -5)$

47. (a) : Solving $y = 1$ and $y^2 = x$, we get $A \equiv (1, 1)$ and solving $y = -2$ and $y^2 = x$, we get $B \equiv (4, -2)$

$\because$ After reflection both reflected rays pass through focus of the parabola $y^2 = x$

i.e., $C \equiv \left(\frac{1}{4}, 0\right)$ $\therefore$ Required area $= \frac{21}{8}$ sq. units

48. A → (r, s); B → (s); C → (p); D → (q)

A. Any point is $(t, 1 - t)$

The chord with this as mid point

$$y(1-t) - 2a(x+t) = (1-t)^2 - 4at$$

$\Rightarrow$ $(1 - t)^2 = 2a(1 - a) \geq 0 \Rightarrow 0 < a \leq 1$

$\therefore$ Length of latus Rectum $\in (0, 4]$

B. $P \equiv (1, 0)$, $Q \equiv (4, 0)$

$\therefore$ Equation of circle passes through two points is $(x - 1)(x - 4) + y^2 + \lambda y = 0$

$\therefore$ Length of the tangent from $(0, 0)$ is $\sqrt{4}$ *i.e.*, 2

C. $m_1m_2 = -1$

D. $1 - |h| > 0 \Rightarrow -1 < h < 1$

◆◆

VITEEE 2021

The Vellore Institute of Technology conducts the VIT engineering entrance exam (VITEEE) annually at a national level in a computer based test (CBT) mode. The candidates who crack this examination will get admission in UG engineering courses offered by the institute.

Important Dates

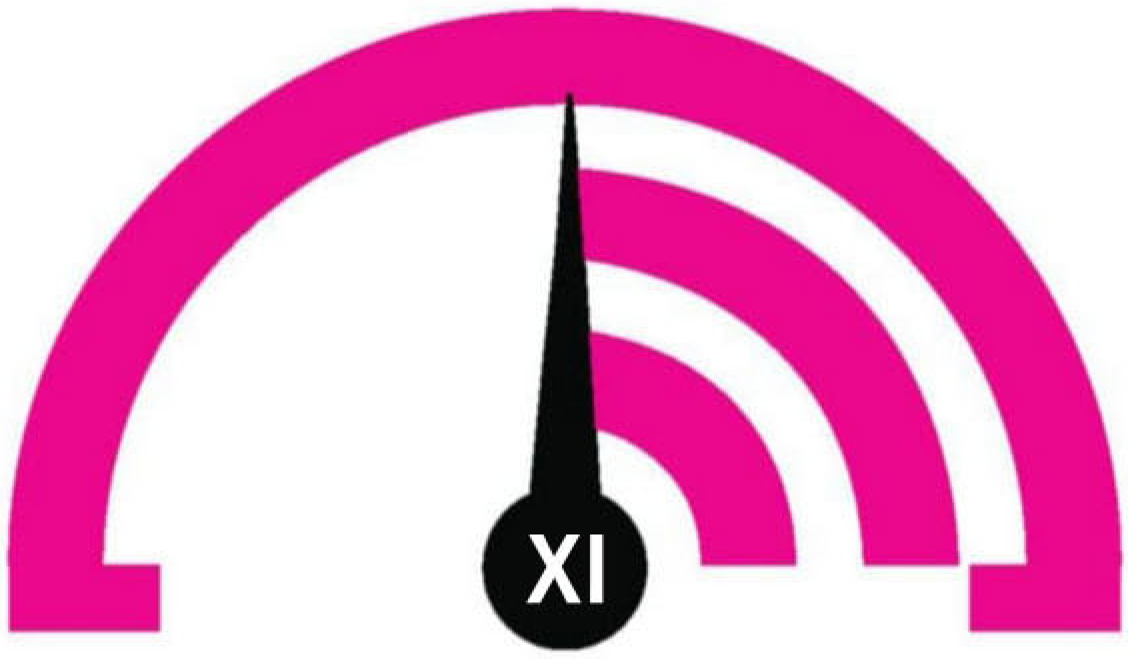
Launch of web application : 30th November, 2020
Last date to apply : 30th March, 2021
Tentative dates for VITEEE-21 : April 2nd to 3rd week, 2021
Counselling date (Tentative) : May 1st week, 2021
Class commencement (Tentative) : July 2nd week, 2021

To apply, candidates must be 10+2 passed/appearing in 2021. The marks needed in 10+2 is 60 percent aggregate in PCM / PCB. However, for the SC/ST candidates and candidates hailing from Jammu and Kashmir / Ladakh, North Eastern states– Arunachal Pradesh, Assam, Manipur, Meghalaya, Mizoram, Nagaland, Sikkim and Tripura– the minimum marks criteria is 50 percent.

The candidates with math, physics, chemistry in 10+2 are eligible for all B.Tech specialisation. Other combinations apply to B.Tech Bio-engineering at VIT Bhopal, B.Tech Biotechnology in VIT Vellore, B.Tech CS with bioinformatics, B.Tech EC with biomedical in VIT Vellore.

For more information visit the official website ***https://viteee.vit.ac.in/.***

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 9 : Conic Sections

Time Taken : 60 Min.

Only One Option Correct Type

1. Equation of the smallest circle passing through the centre of the circle $x^2 + y^2 + 2x + 4y + 4 = 0$ and touching the circle $x^2 + y^2 - 4x - 2y - 3 = 0$ externally is

(a) $x^2 + y^2 - x + y - 4 = 0$
(b) $x^2 + y^2 + x - y - 6 = 0$
(c) $x^2 + y^2 + x + 3y + 2 = 0$
(d) none of these

2. The equation of the parabola whose vertex and focus lie on the axis of x at distances a and a_1 from the origin respectively is

(a) $y^2 = 4(a_1 - a)X$
(b) $y^2 = 4(a_1 - a)(X - a)$
(c) $y^2 = 4(a_1 - a)(X - a_1)$
(d) none of these

3. The curve described parametrically by $x = t^2 + t + 1$, $y = t^2 - t + 1$ represents

(a) a pair of straight line
(b) an ellipse
(c) a parabola
(d) a hyperbola

4. If angle between the tangents at the end points of the chords of the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ is 90°, then locus of the mid points of the chords is

(a) $x^2+y^2=9\left(\frac{x^2}{9}-\frac{y^2}{4}\right)^2$
(b) $x^2+y^2=4\left(\frac{x^2}{9}-\frac{y^2}{4}\right)^2$
(c) $x^2+y^2=5\left(\frac{x^2}{9}-\frac{y^2}{4}\right)^2$
(d) none of these

5. If number of common tangents of the circles $x^2 + y^2 + 2x + 6y + 1 = 0$ and $x^2 + y^2 - 6x + k = 0$ is three, then value of k is.

(a) 2
(b) 4
(c) 6
(d) 5

6. Tangent is drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ $\left(\text{where } \theta\in\left(0,\frac{\pi}{2}\right)\right)$. Then the value of θ such that sum of intercepts on axes made by this tangent is minimum is

(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{6}$
(c) $\frac{\pi}{8}$
(d) $\frac{\pi}{4}$.

One or More Than One Option(s) Correct Type

7. Parabola $y^2 = 4x$ and the circle having its centre at (6, 5) intersect at right angle. Possible point of intersection of these curves can be

(a) (9, 6)
(b) $(2, \sqrt{8})$
(c) (4, 4)
(d) $(3, 2\sqrt{3})$

8. If the tangent at the point $P(\theta)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x = 15$, then θ is equal to

(a) $\frac{2\pi}{3}$
(b) $\frac{4\pi}{3}$
(c) $\frac{5\pi}{3}$
(d) $\frac{\pi}{3}$

9. If the foci of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ coincide with the foci of $\frac{x^2}{25}+\frac{y^2}{9}=1$ and eccentricity of the hyperbola is 2, then

(a) $a^2 + b^2 = 16$
(b) there is no director circle of the hyperbola
(c) centre of the director circle is (0, 0)
(d) length of latus rectum of the hyperbola = 12

10. At a point P on the parabola $y^2 = 4ax$, tangent and normal are drawn. Tangent intersects the x-axis at Q and normal intersects the curve at R such that chord PR subtends an angle of 90° at its vertex. Then

(a) $PQ=2a\sqrt{6}$ (b) $PR=6a\sqrt{3}$

(c) area of $\Delta PQR=18\sqrt{2}a^2$

(d) $PQ=3a\sqrt{2}$

11. P and Q are two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentric angles are differ by 90°, then

(a) locus of point of intersection of tangents at P and Q is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$

(b) locus of mid-point (P, Q) is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$

(c) product of slopes of OP and OQ where O is the centre is $\frac{-b^2}{a^2}$

(d) max. area of ΔOPQ is $\frac{1}{2}ab$

12. The director circle of a hyperbola is $x^2+y^2-4y=0$. One end of the major axis is $(2, 0)$ then the focus is

(a) $\left(\sqrt{3},2-\sqrt{3}\right)$ (b) $\left(-\sqrt{3},2+\sqrt{3}\right)$

(c) $\left(\sqrt{6},2-\sqrt{6}\right)$ (d) $\left(-\sqrt{6},2+\sqrt{6}\right)$

13. If the ellipse $x^2+2y^2=4$ and the hyperbola $S=0$ have same end points of the latus rectum, then the eccentricity of the hyperbola can be

(a) $\operatorname{cosec}\frac{\pi}{4}$ (b) $\operatorname{cosec}\frac{\pi}{3}$

(c) $2\sin\frac{\pi}{3}+\sin\frac{\pi}{4}$ (d) $\sqrt{2}\sin\frac{\pi}{3}+\sin\frac{\pi}{4}$

Comprehension Type

Tangent to the parabola $y=x^2+ax+1$, at the point of intersection of y-axis also touches the circle $x^2+y^2=r^2$. Also, no point of the parabola is below the x-axis.

14. The radius of circle when a attains its maximum value

(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{1}{\sqrt{5}}$ (c) 1 (d) $\sqrt{5}$

15. The minimum area bounded by the tangent and the coordinates axes is

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Matrix Match Type

16. Match the following columns :

	Column-I		Column-II
P.	The foci of the hyperbola $8x^2-y^2-64x+10y+71=0$ are	**1.**	5
Q.	Director circles of $3x^2+2y^2=6$ and $3x^2-2y^2=6$ are $x^2+y^2=k$, where k is	**2.**	2, 3
R.	The foci of the hyperbola $9x^2-16y^2-36x+96y+36=0$ are	**3.**	(10, 5)
S.	Two circles $x^2+y^2+px+py-7=0$ and $x^2+y^2-10x+2py+1=0$ will orthogonally, then the value p is	**4.**	(2, 8)

	P	Q	R	S
(a)	2	1	3	4
(b)	1	3	4	2
(c)	3	1	4	2
(d)	3	4	1	2

Numerical Value Type

17. Number of distinct normal lines that can be drawn to ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ from the point $P(0, 6)$ is ____.

18. From a point on $y=x+1$ tangents are drawn to $\frac{x^2}{2}-y^2=1$ such that the chord of contact passes through fixed point (x_1, y_1), then $\frac{x_1+y_1}{y_1}$ is _____.

19. If the normals of the parabola $y^2=4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2+(y+2)^2=r^2$, then the value of r^2 is ______.

20. The straight line $2x-3y=1$ divides the circular region $x^2+y^2\le 6$ into two parts.

If $S=\left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$

then the number of points(s) in S lying inside the smaller part is _____.

◆◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……

No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CONCEPT MAP

ELLIPSE

Class XI

Definition

Locus of a point which moves in such a way that the sum of its distances from two fixed points is always constant.

Tangent

Equation of tangent is given by

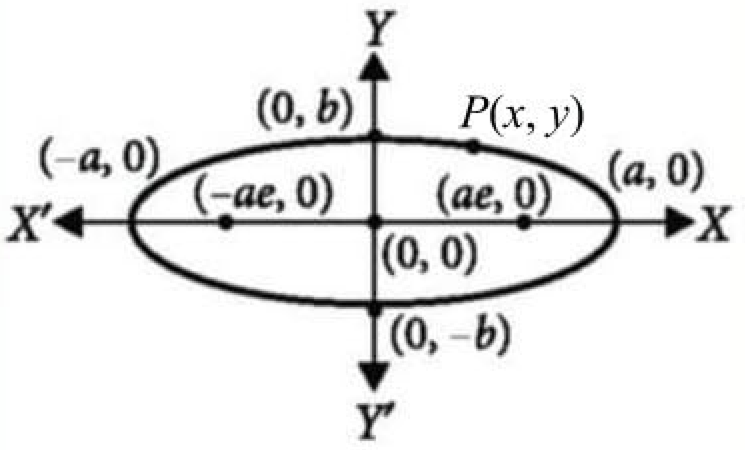
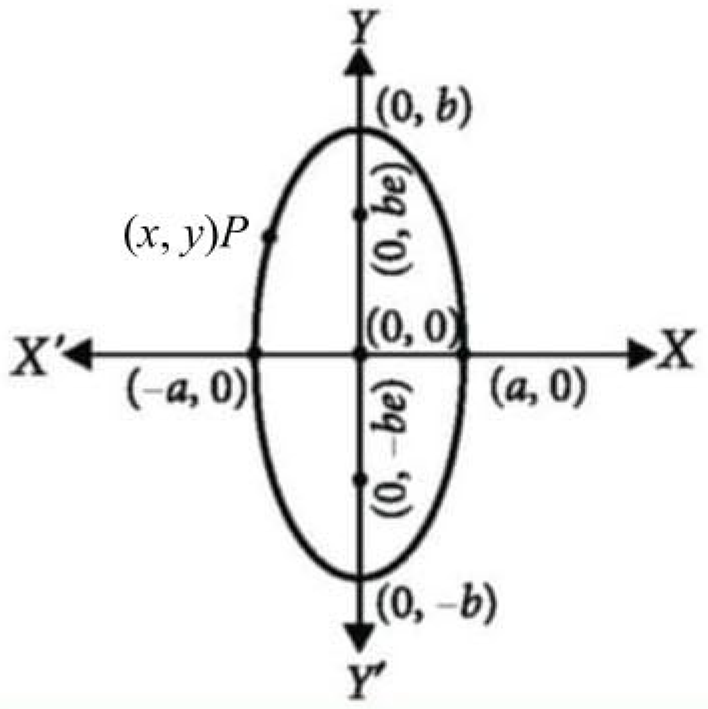
- At (x_1, y_1) : $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$
- At $(a\cos\theta, b\sin\theta)$: $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$
- In slope form : $y = mx \pm \sqrt{a^2m^2 + b^2}$
- Equation of pair of tangents is $S_1^2 = S_{11}S$, where $S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$, $S_1 \equiv \frac{x_1x}{a^2} + \frac{y_1y}{b^2} - 1$, $S_{11} \equiv \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$
- Equation of chord of contact of tangents drawn from a point (x_1, y_1) to ellipse is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

Condition of tangency for a line to the ellipse

Ellipse	Equation of line	Condition
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b)$	$y = mx + c$	$c^2 = a^2m^2 + b^2$
	$x\cos\alpha + y\sin\alpha = p$	$p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$
	$lx + my + n = 0$	$n^2 = a^2l^2 + b^2m^2$

Standard Form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b$
Graph	 Y (0, b) P(x, y) (–a, 0) (a, 0) (–ae, 0) (ae, 0) X′ X (0, 0) (0, –b) Y′	 Y (0, b) (x, y)P (0, be) (0, 0) X′ X (–a, 0) (a, 0) (0, –be) (0, –b) Y′
Centre	(0, 0)	(0, 0)
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Length of major axis	$2a$	$2b$
Length of minor axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x = \pm\left(\frac{a}{e}\right)$	$y = \pm\left(\frac{b}{e}\right)$
Eccentricity	$e = \sqrt{1 - \frac{b^2}{a^2}}$	$e = \sqrt{1 - \frac{a^2}{b^2}}$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

Normal

Equation of normal is given by

At (x_1, y_1) : $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

At $(a\cos\theta, b\sin\theta)$:

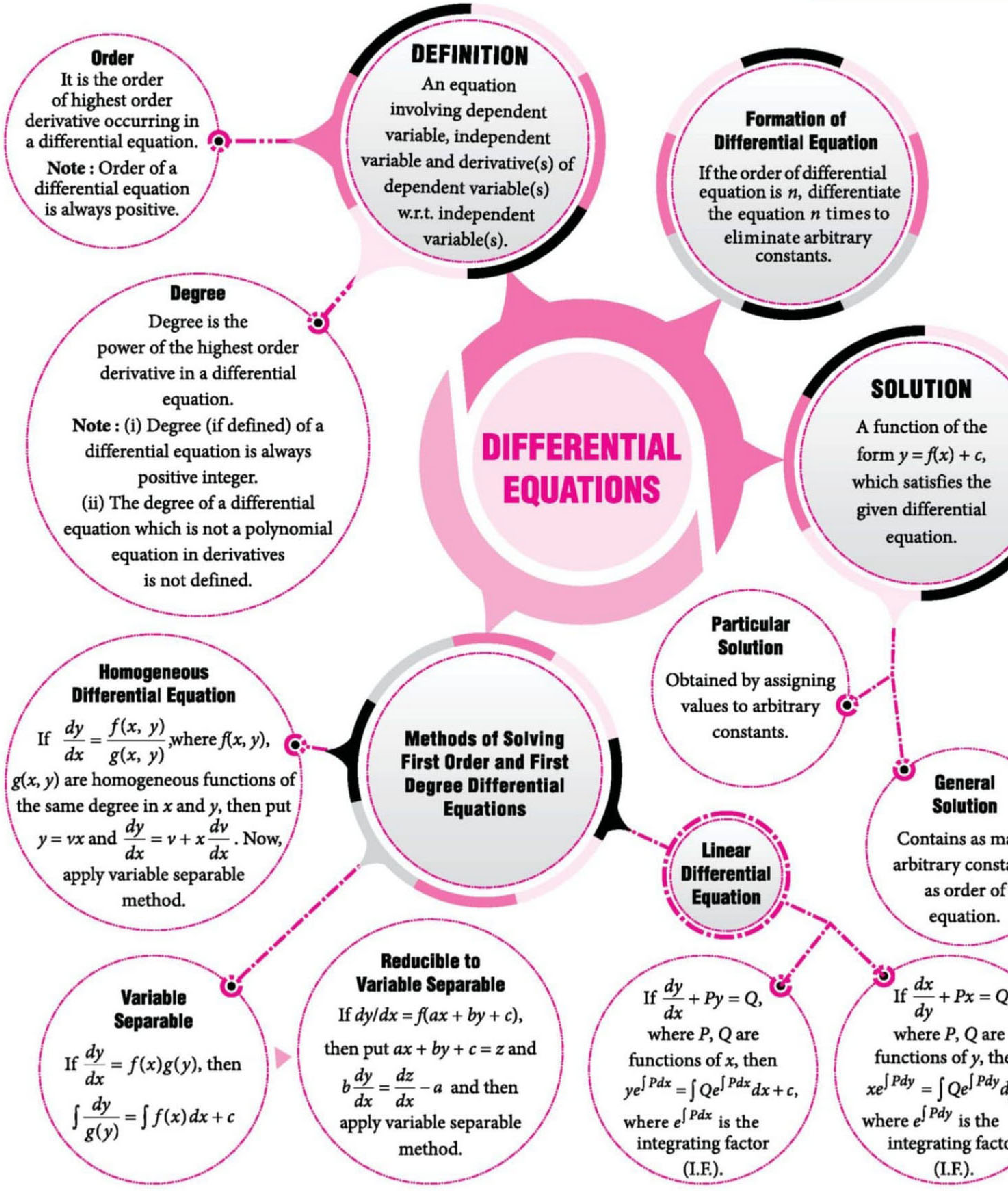
$ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2$

Slope form : $y = mx \pm \frac{m(a^2 - b^2)}{\sqrt{a^2 + b^2m^2}}$

DIFFERENTIAL EQUATIONS

CONCEP
MA

Class XII



DIFFERENTIAL EQUATIONS

DEFINITION

An equation involving dependent variable, independent variable and derivative(s) of dependent variable(s) w.r.t. independent variable(s).

Order

It is the order of highest order derivative occurring in a differential equation.

Note : Order of a differential equation is always positive.

Degree

Degree is the power of the highest order derivative in a differential equation.

Note : (i) Degree (if defined) of a differential equation is always positive integer.
(ii) The degree of a differential equation which is not a polynomial equation in derivatives is not defined.

Formation of Differential Equation

If the order of differential equation is n, differentiate the equation n times to eliminate arbitrary constants.

SOLUTION

A function of the form $y = f(x) + c$, which satisfies the given differential equation.

Particular Solution

Obtained by assigning values to arbitrary constants.

General Solution

Contains as ma
arbitrary consta
as order of
equation.

Methods of Solving First Order and First Degree Differential Equations

Homogeneous Differential Equation

If $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$, where $f(x, y)$, $g(x, y)$ are homogeneous functions of the same degree in x and y, then put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$. Now, apply variable separable method.

Variable Separable

If $\frac{dy}{dx} = f(x)g(y)$, then $\int \frac{dy}{g(y)} = \int f(x)\,dx + c$

Reducible to Variable Separable

If $dy/dx = f(ax + by + c)$, then put $ax + by + c = z$ and $b\frac{dy}{dx} = \frac{dz}{dx} - a$ and then apply variable separable method.

Linear Differential Equation

If $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$, where $e^{\int Pdx}$ is the integrating factor (I.F.).

If $\frac{dx}{dy} + Px = Q$
where P, Q are
functions of y, the
$xe^{\int Pdy} = \int Qe^{\int Pdy} d$
where $e^{\int Pdy}$ is the
integrating facto
(I.F.).

CBSE warm-up!

CLASS-XI

Practice questions for CBSE Exams as per the reduced syllabus
and marking scheme issued by CBSE for the academic session 2020-21.

Practice Paper 2021

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(1) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
(2) Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
(3) Both Part A and Part B have internal choices.

Part – A:

(1) It consists of two sections-I and II.
(2) Section I comprises of 16 very short answer type questions.
(3) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

(1) It consists of three sections-III, IV and V.
(2) Section III comprises of 10 questions of 2 marks each.
(3) Section IV comprises of 7 questions of 3 marks each.
(4) Section V comprises of 3 questions of 5 marks each.
(5) Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART-A

SECTION - I

All questions are compulsory. In case of internal choices attempt any one.

1. Determine $P(A)$ for set $A = \{\{2, 1\}, 0\}$.

OR

Write the set $\{3, 9, 27, 81\}$ in set builder form.

2. Find the value of $\sin\left(-\frac{11\pi}{3}\right)$.

3. Find the value of $i^4 + i^5 + i^6 + i^7$.

4. Solve $x^2 + 3 = 0$.

OR

Find the solution of equation $x^2 + x + 1 = 0$.

5. Solve $14x > 72$ where x is natural number.

6. Show that : $\frac{12!}{6!} = 1\cdot3\cdot5\cdot7\cdot9\cdot11\cdot2^6$.

OR

How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

7. If $C(n, 8) = C(n, 6)$, find $C(n, 2)$.

8. Evaluate : $\lim\limits_{x \to -1} \dfrac{x^{10} + x^5 + 1}{x - 1}$

OR

Evaluate : $\lim\limits_{x \to 0} \dfrac{\cos ax}{\cos bx}; a, b \neq 0.$

9. Find the n^{th} term and 19^{th} term of the sequence 5, 2, –1, –4, ...

10. Convert the equation $3x + 4y - 24 = 0$ into intercept form.

11. What is the eccentricity of the curve $4x^2 + y^2 = 100$?

12. Find the distance of point (a, b, c) from y-axis.

13. Find the G.M. between 4 and 9.

OR

Find the sum to infinity for $5, \frac{5}{3}, \frac{5}{9}, ...$

14. Evaluate : $\dfrac{d}{dx} \sin^n x.$

15. In a single throw of a die, find the probability of getting an even prime number.

16. Find the range of the following set of observations 2, 3, 5, 9, 8, 7, 6, 5, 7, 4, 3.

SECTION - II

Both the case study based questions are compulsory. Attempt any 4 sub-parts from each question 17 and 18. Each sub-part carries 1 mark.

17. If $z = x + iy$, $x, y \in R$, then
$e^z = e^x(\cos y + i \sin y)$,
$\sin z = \dfrac{1}{2i}\left(e^{iz} - e^{-iz}\right)$ and $\cos z = \dfrac{1}{2}\left(e^{iz} + e^{-iz}\right)$.

(i) Number of value of z for which $e^z = 0$ is
(a) 0 (b) 1 (c) 2 (d) ∞

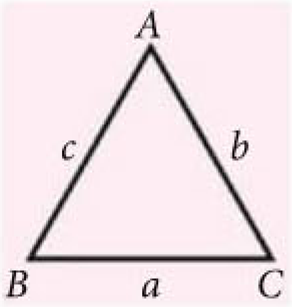
(ii) Number of values of z for which $\sin z = 0$ is
(a) 0 (b) 1 (c) 2 (d) ∞

(iii) Number of values of z for which $\cos z = 0$ is
(a) 0 (b) 1 (c) 2 (d) ∞

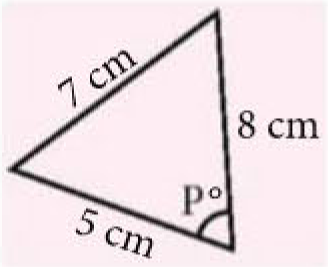
(iv) Solution set of $\sin(iy) = 0$ is
(a) $n\pi, n \in I$ (b) 0
(c) $(2n-1)\dfrac{\pi}{2}, n \in I$ (d) none of these

(v) The value of $\sin^2 z + \cos^2 z$ equals to
(a) 0 (b) 1
(c) –1 (d) none of these

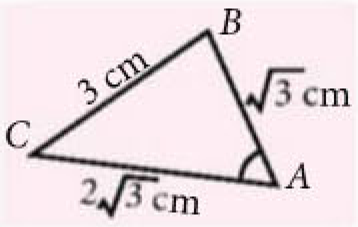
18. In ΔABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ and $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, where a, b, c are the sides and A, B, C are the angles.



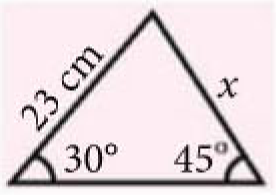
(i) If sides of the triangle are 5 cm, 8 cm and 7 cm shown in figure, then find the value of P.
(a) 30 (b) 60
(c) 45 (d) 75



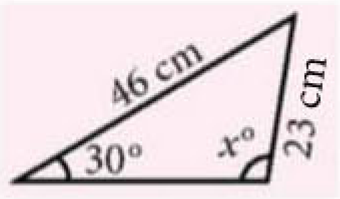
(ii) If sides of the triangle ABC are 3 cm, $\sqrt{3}$ cm, $2\sqrt{3}$ cm then find the value of $\angle A$.
(a) 30° (b) 45°
(c) 60° (d) 75°



(iii) In the given figure, find the value of x.
(a) 23 cm
(b) $23\sqrt{2}$ cm
(c) 46 cm
(d) none of these



(iv) Find the value of x in the given figure.
(a) 110 (b) 120
(c) 105 (d) 90



(v) If the sides of a triangle are 8 cm , 10 cm and 12 cm, then greatest angle is
(a) twice the smallest angle
(b) thrice the smallest angle
(c) half of the smallest angle
(d) none of these

PART-B

SECTION - III

All questions are compulsory. In case of internal choices attempt any one.

19. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinate of the other end of the diameter.

20. If $g(x) = x^2 - 10x - 18$. Determine the image of –2 and the pre-image of 6.

OR

If $A = \{p, q, r\}$, $B = \{r, s\}$ and $C = \{p, t\}$, then verify $A \times (B \cap C) = (A \times B) \cap (A \times C)$

21. Find x and y if $(3x - 2iy)(2 + i)^2 = 10(1 + i)$.

OR

Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$.

22. Prove that :
$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

23. Find the distance between the parallel lines $y = mx + c$ and $y = mx + c'$.

24. The English alphabet has 5 vowels and 21 consonants, how many words with two different vowels and two different consonants can be formed from alphabet?

25. Evaluate the following limit.

$$\lim_{x\to 0} \frac{ax + x\cos x}{b\sin x}$$

OR

If $y = \frac{\cos x}{1+\sin x}$, find $\frac{dy}{dx}$.

26. Show that $A \cap B = A \cap C$ need not imply $B = C$.

27. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

28. In a G.P., the 3rd term is 24 and 6th term is 192. Find the tenth term.

SECTION - IV

All questions are compulsory. In case of internal choices attempt any one.

29. Prove that : $4\sin x \sin\left(x+\frac{\pi}{3}\right)\sin\left(x+\frac{2\pi}{3}\right) = \sin 3x$

30. Differentiate $\frac{2x^3 - \sin x}{\cot x}$ w.r.t. x.

31. If the letters of the word 'PATTERN' are written in all possible orders and are arranged in dictionary order. What is the rank of the word 'PATTERN'?

OR

There are 15 points in a plane out of which only 6 are in a straight line.

(i) How many different straight lines can be made?

(ii) How many triangles can be made?

32. Let S_n denote the sum of the first n terms of an A.P.

If $S_{2n} = 5S_n$, then prove that $\frac{S_{6n}}{S_{3n}} = \frac{17}{4}$.

33. From a group of 2 boys and 3 girls, two children are selected at random. Describe the events:

(i) A : Event that both the selected children are girls

(ii) B : Event that the selected group consists of one boy and one girl

(iii) C : Event that at least one boy is selected

Which pairs of events are mutually exclusive?

34. Find the equation of the parabola whose focus is at $(-1, -2)$ and the directrix is the line $x - 2y + 3 = 0$.

OR

Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13.

35. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

SECTION - V

All questions are compulsory. In case of internal choices attempt any one.

36. Find the domain and range of the following real valued functions:

(i) $f(x) = \frac{3x}{8-x}$ (ii) $f(x) = \frac{1}{\sqrt{x^2 - 49}}$

OR

If $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$. Let R be a relation on A defined by $\{(x, y) : x \in A, y \in A$ and x divides $y)$.

(a) Draw arrow diagram of R.

(b) Find

(i) R in roster form (ii) Domain of R

(iii) Range of R

37. Solve the equation : $2|z|^2 + z^2 - 5 + i\sqrt{3} = 0$

OR

Find the magnitude and conjugate of the number $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$.

38. Calculate the mean and standard deviation of the following data :

Age	No. of persons
20 – 30	3
30 – 40	51
40 – 50	122
50 – 60	141
60 – 70	130
70 – 80	51
80 – 90	2

OR

Find mean and mean deviation about median for the following data :

Class Interval	Frequency
0 – 10	6
10 – 20	7
20 – 30	15
30 – 40	16
40 – 50	4
50 – 60	2

SOLUTIONS

1. $P(A) = \{\phi, \{2\}, \{\{1, 0\}\}, \{2, \{1, 0\}\}\}$

OR

Let $A = \{3, 9, 27, 81\}$

Thus, $A = \{x : x = 3^n, n \in N \text{ and } 1 \le n \le 4\}$

2. $$\sin\left(-\frac{11\pi}{3}\right) = -\sin\left(\frac{11\pi}{3}\right) = -\sin\left(4\pi - \frac{\pi}{3}\right)$$
$$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

3. We have, $i^4 + i^5 + i^6 + i^7 = i^4[1 + i + i^2 + i^3]$

$= 1[1 + i - 1 - i]$ $\quad [\because i^4 = 1, i^2 = -1, i^3 = -i]$

$= 0$

4. Given, $x^2 + 3 = 0 \Rightarrow x^2 = -3$

$\Rightarrow \quad x = \pm\sqrt{-3} = \pm\sqrt{3}i$

OR

Given, $x^2 + x + 1 = 0$

$$\therefore \quad x = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{-1 \pm \sqrt{-3}}{2}$$
$$= \frac{-1 \pm \sqrt{3}i}{2} = \frac{-1}{2} \pm \frac{\sqrt{3}i}{2}$$

5. We have, $14x > 72 \Rightarrow x > 5\frac{1}{7}$

Since, $x \in N$. So, $x = 6, 7, \ldots\ldots$

6. $$\frac{12!}{6!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

$= 12 \times 11 \times 10 \times 9 \times 8 \times 7 = 1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 2^6$

OR

3-digit numbers are to be formed using digits 1 to 9 with no digit repeated.

This can be done in $9 \times 8 \times 7 = 504$ ways.

7. We have, ${}^nC_8 = {}^nC_6 \Rightarrow n = 6 + 8 = 14$

$$\therefore C(n, 2) = C(14, 2) = {}^{14}C_2 = \frac{14!}{2!12!} = 91$$

8. $$\lim_{x \to -1} \frac{x^{10} + x^5 + 1}{x - 1} = \frac{1 - 1 + 1}{-1 - 1} = -\frac{1}{2}$$

OR

$$\lim_{x \to 0} \frac{\cos ax}{\cos bx} = \frac{1}{1} = 1$$

9. Clearly, the given sequence is an A.P. with $a = 5$ and $d = -3$.

$\therefore \quad t_n = a + (n - 1)d = 5 + (n - 1)(-3) = -3n + 8.$

For the 19th term, putting $n = 19$, we get

$t_{19} = (-3)(19) + 8 = -49.$

10. The given equation is $3x + 4y - 24 = 0$

Dividing both sides by 24, we get

$$\frac{3x}{24} + \frac{4y}{24} = 1 \Rightarrow \frac{x}{8} + \frac{y}{6} = 1$$

11. Given, $4x^2 + y^2 = 100 \Rightarrow \frac{x^2}{25} + \frac{y^2}{100} = 1$

Here, $a^2 = 100$, $b^2 = 25$

$$\therefore \quad c = \sqrt{a^2 - b^2} = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$$

$$\therefore \quad e = \frac{c}{a} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}.$$

12. The coordinates of foot of perpendicular from point (a, b, c) on y-axis are $(0, b, 0)$.

$$\therefore \text{ Required distance } = \sqrt{(0 - a)^2 + (b - b)^2(0 - c)^2}$$
$$= \sqrt{a^2 + c^2} \text{ units.}$$

13. The G.M. between 4 and 9 is given by

$G = \sqrt{4 \times 9} = 6.$

OR

Here $a = 5$, $r = \frac{1}{3}$

$$\therefore \quad S_\infty = \frac{a}{1 - r} = \frac{5}{1 - \frac{1}{3}} = \frac{15}{2} = 7.5$$

14. $$\frac{d}{dx}(\sin^n x) = \frac{d}{dx}(\sin x)^n$$
$$= n(\sin x)^{n-1} \cdot \frac{d}{dx}(\sin x)$$
$$\therefore \quad \frac{d}{dx}(\sin^n x) = n(\sin^{n-1} x) \cdot \cos x$$

15. Let $S = \{1, 2, 3, 4, 5, 6\}$ be the sample space and $E = \{2\}$ be event of getting an even prime number

$\Rightarrow \quad n(S) = 6$ & $n(E) = 1$

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{1}{6}$

16. Range = Maximum value – least value = 9 – 2 = 7

17. (i) (a) : $e^z = 0 \Rightarrow e^x \cos y = 0, e^x \sin y = 0$

As $e^x \neq 0$, $\cos y = \sin y = 0$, which is not possible.

(ii) (d) : $\sin z = 0 \Rightarrow e^{2iz} = 1 \Rightarrow e^{-2y}(\cos 2x + i \sin 2x) = 1$

$\Rightarrow \quad e^{-2y} \sin 2x = 0$

$\Rightarrow \quad \sin 2x = 0 \Rightarrow x = n\pi/2, n \in I$

Also, $e^{-2y}\cos(n\pi) = 1 \Rightarrow (-1)^n e^{-2y} = 1 \Rightarrow n = 2m$

and $e^{-2y} = 1 \Rightarrow y = 0$

(iii) (d) : Proceed same as in (ii).

(iv) (b) : $\sin(iy) = 0 \Rightarrow e^{-2y} = 1 \Rightarrow y = 0$

(v) (b) : $\sin^2 z + \cos^2 z = \frac{1}{4i^2}(e^{iz} - e^{-iz})^2 + \frac{1}{4}(e^{iz} + e^{-iz})^2$

$= \frac{1}{4}\left[(e^{iz} + e^{-iz})^2 - (e^{iz} - e^{-iz})^2\right]$

$= \frac{1}{4}(2 - (-2)) = \frac{1}{4}(2 + 2) = 1$

18. (i) (b) : Here $a = 7$ cm, $b = 5$ cm, $c = 8$ cm

Using cosine formula, we have

$$\cos P° = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8} = \frac{40}{10 \times 8} = \frac{1}{2} = \cos 60°$$

$\Rightarrow \quad P = 60$

(ii) (c) : Clearly, $\cos A = \frac{(\sqrt{3})^2 + (2\sqrt{3})^2 - 3^2}{2 \cdot \sqrt{3} \cdot 2\sqrt{3}}$

$= \frac{3 + 12 - 9}{12} = \frac{6}{12} = \frac{1}{2} \Rightarrow A = 60°$

(iii) (d) : By sine rule, we have

$$\frac{x}{\sin 30°} = \frac{23}{\sin 45°} \Rightarrow 2x = 23\sqrt{2} \Rightarrow \frac{23\sqrt{2}}{2} \text{ cm}$$

(iv) (d) : By sine rule, we have

$$\frac{46}{\sin x°} = \frac{23}{\sin 30°} \Rightarrow 2 \sin 30° = \sin x°$$

$\Rightarrow \quad 2 \cdot \frac{1}{2} = \sin x° \Rightarrow \sin x° = 1 \Rightarrow x° = 90° \Rightarrow x = 90$

(v) (a) : Let $a = 8$ cm, $b = 10$ cm and $c = 12$ cm.

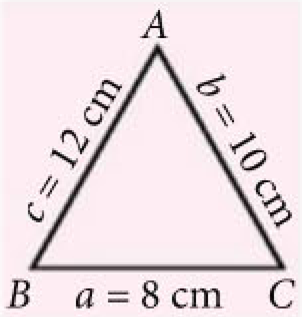
Here A is the smallest angle and C is the greatest angle.

Now, $\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{64 + 100 - 144}{2 \cdot 8 \cdot 10} = \frac{1}{8}$

and $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{100 + 144 - 64}{2 \cdot 10 \cdot 12} = \frac{3}{4}$

$\because \quad \cos 2A = 2\cos^2 A - 1$

$\quad = 2 \times \frac{9}{16} - 1 = \frac{1}{8}$



Thus, $\cos 2A = \frac{1}{8} = \cos C \Rightarrow C = 2A$

19. Given equation of the circle is

$x^2 + y^2 - 4x - 6y + 11 = 0$ ...(i)

Compare (i) with general equation

$x^2 + y^2 + 2gx + 2fy + c = 0$ of circle, we get

$2g = -4$ and $2f = -6$

$\Rightarrow \quad$ Centre $(-g, -f) = (2,3)$

Let the co-ordinates of the other end of diameter of the circle are (x, y). Now, $(2, 3)$ is the mid-point of diameter.

$\therefore \quad 2 = \frac{3 + x}{2} \Rightarrow x = 1$ & $3 = \frac{4 + y}{2} \Rightarrow y = 2$

Hence, co-ordinates of the other end of the diameter of circle are (1,2).

20. Here, $g(x) = x^2 - 10x - 18$

Image of -2 *i.e.*, $g(-2) = (-2)^2 - 10(-2) - 18$

$= 4 + 20 - 18 = 6$

Now, for pre-image of 6, we have

$x^2 - 10x - 18 = 6 \Rightarrow x^2 - 10x - 24 = 0$

$\Rightarrow \quad (x + 2)(x - 12) = 0 \Rightarrow x = -2, 12$

Hence, -2 and 12 are the pre-image of 6.

OR

We have, $A = \{p, q, r\}$, $B = \{r, s\}$ and $C = \{p, t\}$

$\therefore \quad B \cap C = \phi$

So, L.H.S. $= A \times (B \cap C) = A \times \phi = \phi$

Also, $A \times B = \{(p, r), (p, s), (q, r), (q, s), (r, r), (r, s)\}$

$A \times C = \{(p, p), (p, t), (q, p), (q, t), (r, p), (r, t)\}$

$\therefore \quad$ R.H.S. $= (A \times B) \cap (A \times C) = \phi =$ L.H.S.

21. Given $(3x - 2iy)(2 + i)^2 = 10(1 + i)$

$\Rightarrow \quad (3x - 2iy)(4 + 4i + i^2) = 10 + 10i$

$\Rightarrow \quad (3x - 2iy)(3 + 4i) = 10 + 10i$

$\Rightarrow \quad (9x - 6yi + 12xi - 8i^2y) = 10 + 10i$

$\Rightarrow \quad 9x + 8y + i(12x - 6y) = 10 + i \cdot 10$

Equating real and imaginary parts, we get

$9x + 8y = 10$ ...(i)

and $12x - 6y = 10$ ...(ii)

Solving (i) and (ii), we get $x = \frac{14}{15}, y = \frac{1}{5}$

OR

Given, $|1 - i|^x = 2^x$

$\therefore \quad \left(\sqrt{1^2 + (-1)^2}\right)^x = 2^x$

$\Rightarrow \quad (\sqrt{2})^x = 2^x \Rightarrow 2^{x/2} = 2^x$

$\Rightarrow \quad \frac{x}{2} = 0 \quad \therefore x = 0$

22. Consider, $\cot 3x = \cot(2x + x) = \frac{\cot 2x \cot x - 1}{\cot 2x + \cot x}$

$\Rightarrow \quad \cot 3x \cot 2x + \cot 3x \cot x = \cot 2x \cot x - 1$

$\Rightarrow \quad \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

23. The given lines are $mx - y + c = 0$ ...(i)

and $mx - y + c' = 0$ ...(ii)

Let $P(x_1, y_1)$ be any point on (i), so that

$$mx_1 - y_1 + c = 0 \text{ or } mx_1 - y_1 = -c \qquad \text{...(iii)}$$

Distance between parallel lines (i) and (ii)

= Distance of $P(x_1, y_1)$ from (ii)

$$= \left|\frac{mx_1 - y_1 + c'}{\sqrt{m^2+1}}\right| = \left|\frac{-c+c'}{\sqrt{m^2+1}}\right| \text{ units} \qquad \text{[Using (iii)]}$$

24. We have 5 vowels and 21 consonants as our english alphabets.

$\therefore$ Number of words formed from 2 different vowels and 2 different consonants = ${}^5C_2 \times {}^{21}C_2$

= 10 × 210 = 2100

Now, 4 selected letters can arrange themselves in 4! ways

$\therefore$ Total words formed 2100 × 4! = 50400

25. Given, $\lim\limits_{x\to 0} \dfrac{ax + x\cos x}{b\sin x} = \lim\limits_{x\to 0} \dfrac{x(a+\cos x)}{b\sin x}$

$$= \lim_{x\to 0} \frac{(a+\cos x)}{b\left(\dfrac{\sin x}{x}\right)} = \frac{a+\cos 0}{b\times 1} = \frac{a+1}{b}$$

OR

We have, $y = \dfrac{\cos x}{1+\sin x}$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

$$= \frac{-\sin x - 1}{(1+\sin x)^2} = \frac{-(1+\sin x)}{(1+\sin x)^2} = \frac{-1}{1+\sin x}$$

26. Let $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5, 6\}$, $C = \{2, 3, 4, 9, 10\}$.

$\therefore$ $A \cap B = \{1, 2, 3, 4\} \cap \{2, 3, 4, 5, 6\} = \{2, 3, 4\}$

$A \cap C = \{1, 2, 3, 4\} \cap \{2, 3, 4, 9, 10\} = \{2, 3, 4\}$

Now we have $A \cap B = A \cap C$. But $B \neq C$.

27. Let x be the smaller of two positive consecutive even integers, then the other one is $x + 2$.

We have, $x > 5$ and $x + x + 2 < 23 \Rightarrow 2x + 2 < 23$

$\Rightarrow 2x < 21 \Rightarrow x < 21/2$

$\therefore$ x can be 6, 8, 10

($\because$ $x > 5$ and numbers are even integers)

$\therefore$ The pairs are [(6, 8), (8, 10), (10, 12)]

28. Let the first term of G.P. be a and the common ratio be r.

Given, $T_3 = 24 \Rightarrow ar^2 = 24$ ...(i)

and $T_6 = 192 \Rightarrow ar^5 = 192$ ...(ii)

Dividing (ii) by (i), we get

$$\frac{ar^5}{ar^2} = \frac{192}{24} \Rightarrow r^3 = 8 \Rightarrow r = 2$$

Putting $r = 2$ in (i), we get $a(2)^2 = 24 \Rightarrow a = 6$

$\therefore$ $T_{10} = ar^9 = 6(2)^9 = 3072$

29. L.H.S. $= 4\sin x \sin\left(x + \dfrac{\pi}{3}\right)\sin\left(x + \dfrac{2\pi}{3}\right)$

$$= 2\sin x \cdot 2\sin\left(x+\frac{\pi}{3}\right)\sin\left(x+\frac{2\pi}{3}\right)$$

$$= 2\sin x\left(-\cos\left(\frac{2\pi}{3}+x+x+\frac{\pi}{3}\right)+\cos\left(\frac{2\pi}{3}+x-\frac{\pi}{3}-x\right)\right)$$

$$= 2\sin x\left(-\cos(2x+\pi)+\cos\frac{\pi}{3}\right)$$

$$= 2\sin x\left(\frac{1}{2}+\cos 2x\right) = \sin x + 2\sin x\cos 2x$$

$= \sin x + \sin 3x - \sin x = \sin 3x =$ R.H.S.

30. *Refer to Answer 110, page no. 297 of MTG CBSE Champion Mathematics, Class 11.*

31. Total letters in the word 'PATTERN' is 7

$\therefore$ Total no. of words starting with 'A' = $\dfrac{6!}{2!}$

Total no. of words starting with E = $\dfrac{6!}{2!}$

Total no. of words starting with N = $\dfrac{6!}{2!}$

MTG

The only thing you NEED for excellence in Class -11

CBSE CHAMPION CHAPTERWISE-TOPICWISE PHYSICS ₹ 400

CBSE CHAMPION CHAPTERWISE-TOPICWISE CHEMISTRY ₹ 400

CBSE CHAMPION CHAPTERWISE-TOPICWISE MATHEMATICS ₹ 350

CBSE CHAMPION CHAPTERWISE-TOPICWISE BIOLOGY ₹ 450

HIGHLIGHTS

- Important Facts/Formulae & Comprehensive Theory
- Practice Questions, NCERT & Exemplar Problems
- HOTS Questions
- Previous Years' Questions of KVS, NCT, etc.
- Answers as per CBSE Marking Scheme
- 10 Practice Papers with Objective Type Questions

Visit www.mtg.in to buy online!

Total no. of words starting with PAE = $\frac{4!}{2!}$

Total no. of words starting with PAN = $\frac{4!}{2!}$

Total no. of words starting with PAR = $\frac{4!}{2!}$

Total no. of words starting with PATE = 3!
Total no. of words starting with PATN = 3!
Total no. of words starting with PATR = 3!
Total no. of words starting with PATTENR = 1

$\therefore$ Total no. of words before 'PATTERN'

$= 3 \times \frac{6!}{2!} + 3 \times \frac{4!}{2!} + 3 \times 3! + 1 = 1135$

Hence, rank of word 'PATTERN' = 1135 + 1 = 1136

OR

(i) Number of straight lines formed joining the 15 points, taking 2 at a time = ${}^{15}C_2 = 105$

Number of straight lines formed by joining the 6 points, taking 2 at a time = ${}^{6}C_2 = 15$

But, 6 collinear points when joined pairwise give only line

$\therefore$ Required number of straight lines = 105 – 15 + 1 = 91

(ii) Number of triangles formed by joining the points, taking 3 at a time = ${}^{15}C_3 = 455$

Number of triangles formed by joining the 6 points, taken 3 at a time = ${}^{6}C_3 = 20$

But, 6 collinear points cannot form a triangle when taken 3 at a time.

So, Required number of triangles = 455 – 20 = 435.

32. Let a be the first term and d be the common difference of the given A.P. Then

$S_{2n} = 5S_n$

$\Rightarrow \frac{2n}{2}[2a+(2n-1)d] = 5.\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow 2[2a + (2n - 1)d] = 5[2a + (n - 1)d]$

$\Rightarrow 4a + 4nd - 2d = 10a + 5nd - 5d$

$\Rightarrow 6a = 3d - nd \Rightarrow 6a = (3 - n)d$

Now $\frac{S_{6n}}{S_{3n}} = \frac{\frac{6n}{2}[2a+(6n-1)d]}{\frac{3n}{2}[2a+(3n-1)d]}$

$= \frac{6[2a+(6n-1)d]}{3[2a+(3n-1)d]} = \frac{12a+36nd-6d}{6a+9nd-3d}$

$= \frac{2(3-n)d+36nd-6d}{(3-n)d+9nd-3d}$

$= \frac{6d-2nd+36nd-6d}{3d-nd+9nd-3d} = \frac{34nd}{8nd} = \frac{17}{4}$

33. Let us name the boys as B_1 and B_2, and the girls as G_1, G_2 and G_3.

Then, $S = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_1G_2, G_1G_3, G_2G_3\}$.

We have, (i) $A = \{G_1G_2, G_1G_3, G_2G_3\}$

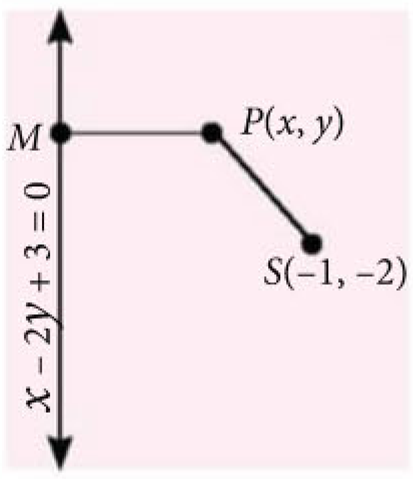
(ii) $B = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

(iii) $C = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, B_1B_2\}$

Clearly, $A \cap B = \phi$ and $A \cap C = \phi$.

Hence, (A, B) and (A, C) are mutually exclusive events.

34. Let $P(x, y)$ be any point on the parabola whose focus is $S(-1, -2)$ and the directrix $x - 2y + 3 = 0$. Draw PM perpendicular from $P(x, y)$ on the directrix $x - 2y + 3 = 0$.



Then, by definition

$SP = PM \Rightarrow SP^2 = PM^2$

$\Rightarrow (x+1)^2+(y+2)^2 = \left(\frac{x-2y+3}{\sqrt{1+4}}\right)^2$

$\Rightarrow 5[(x + 1)^2 + (y + 2)^2] = (x - 2y + 3)^2$

$\Rightarrow 5(x^2 + y^2 + 2x + 4y + 5) = (x^2 + 4y^2 + 9 - 4xy + 6x - 12y)$

$\Rightarrow 4x^2 + y^2 + 4xy + 4x + 32y + 16 = 0$ is the equation of the required parabola.

OR

Refer to Answer 113, page no. 260 of MTG CBSE Champion Mathematics, Class 11.

35. Here $A \cup X = B \cup X$ for some set X

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow A = (A \cap B) \cup (A \cap X) \quad [\because A \cap (A \cup X) = A]$

$\Rightarrow A = (A \cap B) \cup \phi \Rightarrow A = A \cap B \quad$ ...(i)

Also $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = B \quad [\because B \cap (B \cup X) = B]$

$\Rightarrow (B \cap A) \cup \phi = B \quad [\because B \cap X = \phi]$

$\Rightarrow B \cap A = B \quad$ ...(ii)

From (i) and (ii), we get, $A = B$.

36. (i) Here, $f(x) = \frac{3x}{8-x}$

$f(x)$ is not defined for all real values of x except

$8 - x = 0$ *i.e.*, $x = 8$

$\therefore$ Domain of f is $R - \{8\}$

Let $y = f(x)$

mtG

INTRODUCING

THE SMART STUDY SYSTEM

100% CBSE MTG's 100 Percent Series is based on the most current CBSE guidelines and curriculum

100% SCIENTIFIC MTG's editorial team knows what it takes to score more in exams - it keeps a close watch on changing examination patterns and has been able to put all ingredients critical to success together in a package aptly titled 100 Percent

100% CONTENT Be it easy-to-comprehend text, graphics, illustrations or concept maps that complement, understanding theory and concepts is so straightforward with 100 Percent

100% CHECKS After each topic, in every chapter, 100 Percent presents students with a variety of Q&A, including "Try Yourself", "NCERT Focus", "CBSE Focus" & "Competition Focus"

100% PRACTICE 100 Percent goes all out to ensure students are prepared for a diverse set of future challenges through a variety of exam drills and practice papers, even Viva Voce Q&A for lab-based experiments

100 PERCENT SCIENCE
10
100 PERCENT MATHEMATICS
10
100 PERCENT ENGLISH
10
100 PERCENT SOCIAL SCIENCE
10
100 PERCENT SCIENCE
9
100 PERCENT MATHEMATICS
9
100 PERCENT ENGLISH
9
100 PERCENT SOCIAL SCIENCE
9

100 PERCENT
for Science, Mathematics English & Social Science

For CBSE Classes 9 & 10

THE MISSION TO MARKS

Scan to buy online
(QR code reader required)

Visit bit.ly/100percent to buy online or to know more. Or email **info@mtg.in** now.

$$\Rightarrow \quad y = \frac{3x}{8-x} \quad \Rightarrow \quad 8y - xy = 3x$$

$$\Rightarrow \quad 8y = x(y+3) \quad \Rightarrow \quad x = \frac{8y}{y+3}, x \in R$$

$$\Rightarrow \quad y + 3 \neq 0 \Rightarrow y \neq -3$$

Hence, range of f is $R - \{-3\}$.

(ii) Here, $f(x) = \dfrac{1}{\sqrt{x^2 - 49}}$

$f(x)$ is defined if $\dfrac{1}{\sqrt{x^2 - 49}}$ is real number

$$\Rightarrow \quad x^2 - 49 > 0 \quad \Rightarrow \quad x^2 > 49 \Rightarrow x < -7, x > 7$$

$\therefore$ Domain $= (-\infty, -7) \cup (7, \infty)$

Let $y = f(x) = \dfrac{1}{\sqrt{x^2 - 49}}, y > 0$

$$\Rightarrow \quad y^2 = \frac{1}{x^2 - 49} \quad \Rightarrow \quad x^2 - 49 = \frac{1}{y^2}$$

$$\Rightarrow \quad x^2 = 49 + \frac{1}{y^2} > 0$$

$\therefore$ Range of $f = R - \{0\}$.

OR

Refer to Answer 63, page no. 35 of MTG CBSE Champion Mathematics, Class 11.

37. Let $z = x + iy$

$\Rightarrow \quad z^2 = x^2 - y^2 + 2ixy$ and $|z|^2 = x^2 + y^2$

Given, $2|z|^2 + z^2 - 5 + i\sqrt{3} = 0$

$$\Rightarrow \quad 2(x^2 + y^2) + x^2 - y^2 + 2ixy - 5 + i\sqrt{3} = 0$$

$$\Rightarrow \quad 2x^2 + 2y^2 + x^2 - y^2 - 5 + i(2xy + \sqrt{3}) = 0 + 0i$$

Equating real and imaginary parts, we get

$3x^2 + y^2 - 5 = 0$ ...(i)

and $2xy + \sqrt{3} = 0 \Rightarrow 2xy = -\sqrt{3}$ ...(ii)

$$\Rightarrow \quad y = \frac{-\sqrt{3}}{2x}$$

Substituting this value of y in (i), we get

$$3x^2 + \frac{3}{4x^2} - 5 = 0$$

$$\Rightarrow \quad 12x^4 - 20x^2 + 3 = 0 \Rightarrow 12x^4 - 18x^2 - 2x^2 + 3 = 0$$

$$\Rightarrow \quad 6x^2(2x^2 - 3) - 1(2x^2 - 3) = 0 \Rightarrow (6x^2 - 1)(2x^2 - 3) = 0$$

$$\Rightarrow \quad x = \pm\frac{1}{\sqrt{6}}, \pm\sqrt{\frac{3}{2}}$$

When $x = \dfrac{1}{\sqrt{6}}$, then $y = \dfrac{-3\sqrt{2}}{2} = \dfrac{-3}{\sqrt{2}}$

When $x = \dfrac{-1}{\sqrt{6}}$, then $y = \dfrac{3\sqrt{2}}{2} = \dfrac{3}{\sqrt{2}}$

When $x = \sqrt{\dfrac{3}{2}}$, then $y = \dfrac{-1}{\sqrt{2}}$

When $x = -\sqrt{\dfrac{3}{2}}$, then $y = \dfrac{1}{\sqrt{2}}$

$\therefore$ Possible values of z are $\dfrac{1}{\sqrt{6}} - i\dfrac{3}{\sqrt{2}}$,

$$\frac{-1}{\sqrt{6}} + i\frac{3}{\sqrt{2}}, -\sqrt{\frac{3}{2}} + i\frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}} - i\frac{1}{\sqrt{2}}$$

OR

Refer to Answer 60, page no. 112 of MTG CBSE Champion Mathematics, Class 11.

38. Here, $h = 10$ and let assumed mean $(A) = 55$

Age	**Mid-values (x_i)**	f_i	$u_i = \dfrac{x_i - A}{h}$	$f_i u_i$	$f_i u_i^2$
20-30	25	3	−3	−9	27
30-40	35	51	−2	−102	204
40-50	45	122	−1	−122	122
50-60	55 = A	141	0	0	0
60-70	65	130	1	130	130
70-80	75	51	2	102	204
80-90	85	2	3	6	18
Total		500		5	705

Here, $\bar{x} = A + \dfrac{\Sigma f_i u_i}{N} \times h = 55 + \dfrac{5}{500} \times 10 = 55.1$

$$\sigma^2 = \left[\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right] \times h^2$$

$$= \left[\frac{705}{500} - \left(\frac{5}{500}\right)^2\right] \times 100$$

$$= 100\left[\frac{705}{500} - \frac{1}{10000}\right]$$

$$= 100\left[\frac{14100 - 1}{10000}\right] = \frac{100 \times 14099}{10000} = 140.99$$

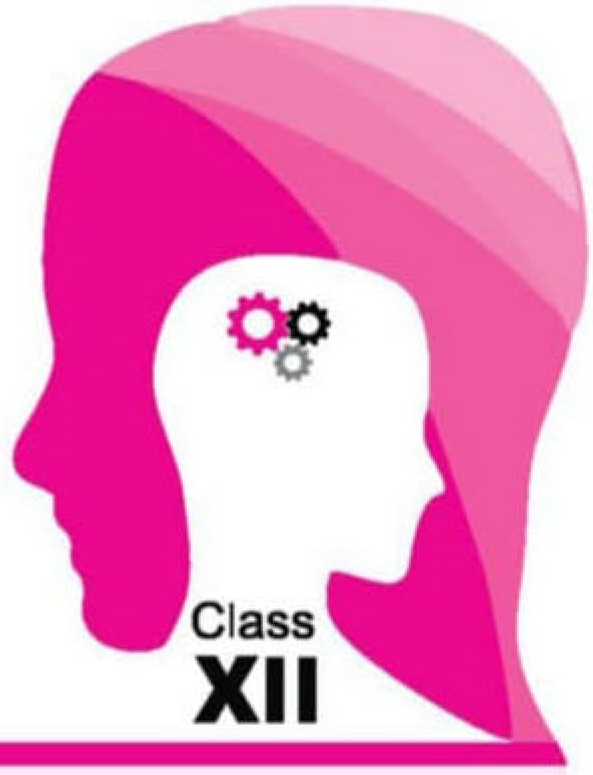
$\therefore$ Standard deviation, $\sigma = \sqrt{140.99} = 11.87$

OR

Refer to Answer 34, page no. 328-329 of MTG CBSE Champion Mathematics, Class 11.

◆◆

CONCEPT BOOSTER



This column is aimed at Class XII students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

***ALOK KUMAR**

Integration

DEFINITION

If $F(x)$ be a differentiable function of x such that $\frac{d}{dx}[F(x)] = f(x)$. Then $F(x)$ is called the integral or anti-derivative or primitive function of the integrand $f(x)$. Symbolically, it is written as $\int f(x)dx = F(x) + c$, where c is called the constant of integration and can take any real value.

BASIC FORMULAE

Anti-derivatives or integrals of some of the widely used functions (integrands) are given below:

- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$
- $\int \frac{1}{x}\,dx = \ln|x| + c$
- $\int e^x\,dx = e^x + c$
- $\int a^x\,dx = \frac{a^x}{\ln a} + c,\ a > 0,\ a \neq 1$
- $\int \cos x\,dx = \sin x + c$
- $\int \sin x\,dx = -\cos x + c$
- $\int \sec^2 x\,dx = \tan x + c$
- $\int \sec x \tan x\,dx = \sec x + c$
- $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$
- $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$
- $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{|x|\sqrt{x^2 - 1}}\,dx = \sec^{-1} x + c$
- $\int \cot x\,dx = \ln|\sin x| + c$
- $\int \tan x\,dx = \ln|\sec x| + c$
- $\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\int \operatorname{cosec} x\,dx = \ln|\cot x - \operatorname{cosec} x| + c$

STANDARD FORMULAE

- $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$
- $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln\left|x + \sqrt{x^2 + a^2}\right| + c$
- $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right| + c$
- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left|x + \sqrt{x^2 - a^2}\right| + c$
- $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left|x + \sqrt{x^2 + a^2}\right| + c$
- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x - a}{x + a}\right| + c$

---

*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

METHODS OF INTEGRATION

In order to find the integral of complex problems, generally three rules of integration are used :

- Integration by substitution.
- Integration by parts.
- Integration by partial fractions.

INTEGRATION BY SUBSTITUTION

Direct Substitution

- If integral is of the form $\int f(g(x))\, g'(x)\, dx$, then put $g(x) = t$, provided $\int f(t)dt$ exists.
- $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$
- $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

Standard Substitutions

Expression	Substitution
$x^2 + a^2$ or $\sqrt{x^2 + a^2}$	$x = a \tan \theta$ or $a \cot \theta$
$x^2 - a^2$ or $\sqrt{x^2 - a^2}$	$x = a \sec \theta$ or $a \operatorname{cosec} \theta$
$a^2 - x^2$ or $\sqrt{a^2 - x^2}$	$x = a \sin \theta$ or $x = a \cos \theta$
$\sqrt{a + x}$ or $\sqrt{a - x}$	$x = a \cos 2\theta$
$\left(x \pm \sqrt{x^2 \pm a^2}\right)^n$	expression inside the bracket = t
$\sqrt{(x - a)(b - x)}$	$x = a \cos^2 \theta + b \sin^2 \theta$
$\frac{1}{(x+b)^{1-\frac{1}{n}}(x+a)^{1+\frac{1}{n}}} (n \in N, n > 1)$	$\frac{x+b}{x+a} = t$

Indirect Substitution

If the integrand is of the form $f(x)\, g(x)$, where $g(x)$ is a function of the integral of $f(x)$, then put integral of $f(x) = t$.

INTEGRATION BY PARTS

If u and v be two functions of x, then integral of product of these two functions is given by

$$\int uv\, dx = u\int v\, dx - \int\left[\frac{du}{dx}\int v\, dx\right]dx$$

In applying the above rule care has to be taken in the selection of the first function (u) and the second function (v). Normally we use the following methods:

- If in the product of the two functions, one of the functions is not directly integrable (*e.g.*, $\ln x$, $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ etc.), then we take it as the first function and the remaining function is taken as the second function.
- If there is no other function, then unity is taken as the second function.
- If both the functions are directly integrable, then the first function is chosen in such a way that the derivative of the function thus obtained under integral sign is easily integrable. Usually we use the following preference order for the first function.
  (Inverse, Logarithmic, Algebraic, Trigonometric, Exponential)
  In the above stated order, the function on the left is always chosen as the first function. This rule is called as ILATE. For example, in $\int x \sin x\, dx$, x is taken as the first function and $\sin x$ is taken as the second function.

INTEGRATION BY PARTIAL FRACTIONS

- A function of the form $P(x)/Q(x)$, where $P(x)$ and $Q(x)$ are polynomials, is called a rational function. Consider the rational function
  $$\frac{x+7}{(2x-3)(3x+4)} = \frac{1}{2x-3} - \frac{1}{3x+4}$$
  The two fractions on the RHS are called partial fractions. To integrate the rational function on the LHS, it is enough to integrate the two fractions on the RHS, which are easily integrable. This is known as method of partial fractions. In case the degree of $P(x)$ (numerator) is not less than that of $Q(x)$ (denominator), we carry out the division of $P(x)$ by $Q(x)$ and reduce the degree of the numerator.
- In order to write $P(x)/Q(x)$ in partial fractions, first of all we write
  $Q(x) = (x - a)^k \ldots (x^2 + \alpha x + \beta)^r \ldots$ where binomials are different, and then set
  $$\frac{P(x)}{Q(x)} = \frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_k}{(x-a)^k}$$
  $$+ \frac{M_1x+N_1}{x^2+\alpha x+\beta} + \frac{M_2x+N_2}{(x^2+\alpha x+\beta)^2} + \ldots + \frac{M_rx+N_r}{(x^2+\alpha x+\beta)^r} + \ldots,$$
  where A_1, A_2, ..., A_k, M_1, M_2, ..., M_r, N_1, N_2, ..., N_r are real constants to be determined. These are determined by reducing both sides of the

above identity to integral form and equating the coefficients of equal powers of x, which gives a system of linear equations in the coefficient. The constants can also be obtained by substituting suitably chosen numerical values of x in both sides of the identity.

- Before proceeding to write a rational function as a sum of partial fractions, we should be taken to ascertain that it is either a proper rational fraction or is rewritten as one.

ALGEBRAIC INTEGRALS

- A rational function $\dfrac{P(x)}{Q(x)}$ is proper if the degree of polynomial $Q(x)$ is greater than the degree of the polynomial $P(x)$. In case the degree of $P(x)$ is greater than or equal to the degree of $Q(x)$, we first write $\dfrac{P(x)}{Q(x)}=h(x)+\dfrac{P_1(x)}{Q(x)}$, where $h(x)$ is a polynomial and $P_1(x)$ is a polynomial of degree less than the degree of polynomial $Q(x)$.

- **Integrals of the type** $\displaystyle\int\frac{px+q}{(ax^2+bx+c)}dx$, $\displaystyle\int\frac{px+q}{\sqrt{ax^2+bx+c}}dx,\ \int(px+q)\sqrt{ax^2+bx+c}\,dx$ **:**

  In these type of integrals, we write $px + q = l$ (differential coefficient of $ax^2 + bx + c$) $+ m$ ...(i) Find values of l and m by comparing the coefficients of x and constant term on both sides of (i). In this way, the integration will reduce to the sum of two integrals which can be integrated easily.

- **Integrals of the type**

  $$\frac{ax^2+bx+c}{px^2+qx+r}\ \text{or}\ \frac{ax^2+bx+c}{\sqrt{px^2+qx+r}}:$$

  In this case substitute $ax^2 + bx + c = M(px^2 + qx + r) + N(2px + q) + R$. M, N and R can be calculated by comparing the coefficients. Then, integration will reduce to integration of three independent functions.

Integration of Irrational Algebraic Fractions

- Irrational functions of $(ax + b)^{1/n}$ and x can be easily evaluated by the substitution, $t^n = ax + b$. Thus,

  $$\int f(x,(ax+b)^{1/n})\,dx=\int f\left(\frac{t^n-b}{a},t\right)\frac{nt^{n-1}}{a}dt$$

- To integrate $\displaystyle\int\frac{dx}{(Ax^2+B)\sqrt{Cx^2+D}}$, we put $x = 1/t$, which will reduce the given integral to $\displaystyle-\int\frac{tdt}{(A+Bt^2)\sqrt{C+Dt^2}}$

  Now, substitute $C + Dt^2 = u^2$, which will reduce it to the form $\displaystyle\int\frac{du}{u^2\pm a^2}$

- To integrate $\displaystyle\int\frac{(ax^2+bx+c)\,dx}{(dx+e)\sqrt{fx^2+gx+h}}$, we write,

  $ax^2 + bx + c = A_1(dx + e)(2fx + g) + B_1(dx + e) + C_1$, where A_1, B_1 and C_1 can be determined.

TRIGONOMETRIC INTEGRALS

Integrals of the form $\int R(\sin x, \cos x)\ dx$

- Here R is a rational function of $\sin x$ and $\cos x$. In this case we take universal substitution $\tan(x/2) = t$.

  So, $\sin x=\dfrac{2t}{1+t^2}$; $\cos x=\dfrac{1-t^2}{1+t^2}$

Integrals of the form

$$\int\frac{p\cos x+q\sin x+r}{a\cos x+b\sin x+c}dx\ \ \textbf{or}\ \ \int\frac{p\cos x+q\sin x}{a\cos x+b\sin x}dx$$

- In this integral express numerator as l (denominator) $+ m$(derivative of denominator) $+ n$ and find the values of l, m, n by comparing the coefficients of $\sin x$, $\cos x$ and constant term and split the integral into sum of three integrals as :

  $$l\int dx+m\int\frac{\text{derivative of denominator}}{\text{denominator}}dx$$
  $$+n\int\frac{dx}{\text{denominator}}$$

- Express numerator as l (denominator) $+ m$(derivative of denominator) and find the values of l and m as above.

Integrals of the form $\int(\sin^m x\cos^n x)\ dx$, where m and $n\in N$

- If one of m and n is odd, then substitute for term of even power.
- If both m and n are odd, substitute either of the term.
- If both m and n are even, use trigonometric identities only.

DEFINITE INTEGRATION

Fundamental Theorem of Integral Calculus (Newton-Leibnitz Formula)

- If $f(x)$ is a continuous function on $[a, b]$ and $F(x)$ is an anti-derivative of $f(x)$ on $[a, b]$ *i.e.*,

  $F'(x) = f(x)\ \forall\ x \in (a, b)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

  Here, a and b are the lower and the upper limits of integration.

Properties of Definite Integration

- $\int_a^b f(x)\,dx = \int_a^b f(t)dt$
- $\int_a^b f(x)\,dx = -\int_b^a f(x)dx$
- $\int_a^b f(x)\,dx = \int_a^c f(x)dx + \int_c^b f(x)dx,\ a < c < b$
- $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)dx$
- $\int_0^a f(x)\,dx = \int_0^{a/2} [f(x) + f(a-x)]dx$

  Also, If $f(x) = f(a - x)$, then $\int_0^a f(x)dx = 2\int_0^{a/2} f(x)dx$

  If $f(x) = -f(a - x)$, then $\int_0^a f(x)dx = 0$
- $\int_{-a}^a f(x)dx = \int_0^a [f(x) + f(-x)]dx$

  Also, $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx, \text{ if } f(x) \text{ is even.} \\ 0\ , \quad\quad \text{ if } f(x) \text{ is odd.} \end{cases}$
- $\int_a^b f(x)dx = (b-a)\int_0^1 f((b-a)x + a)dx$
- If $f(x)$ is a periodic function with period T, then

  $\int_a^{a+nT} f(x)dx = n\int_0^T f(x)dx$, where $n \in I$

Differentiation under the Integral Sign (Leibnitz's Rule)

- If g is continuous on $[a, b]$ and $f_1(x)$ and $f_2(x)$ are differentiable functions whose values lie in $[a, b]$, then

$$\frac{d}{dx}\int_{f_1(x)}^{f_2(x)} g(t)dt = g(f_2(x))f_2'(x) - g(f_1(x))f_1'(x)$$

Definite Integral as Limit of a Sum

- $\int_0^1 f(x)dx = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$

PROBLEMS

Single Correct Answer Type

1. $\int \frac{dx}{\sqrt{(x-a)(b-x)}} =$

(a) $2\sin^{-1}\sqrt{\left(\frac{x-a}{b-a}\right)} + c$ (b) $\sin^{-1}\sqrt{\left(\frac{x-a}{b-a}\right)} + c$

(c) $2\sin^{-1}\sqrt{\left(\frac{x+a}{b-a}\right)} + c$ (d) none of these

2. If $\int x e^x \cos x\, dx = f(x) + c$, then $f(x)$ is equal to

(a) $\frac{e^x}{-2}\{(1-x)\sin x - x\cos x\}$

(b) $\frac{e^x}{2}\{(1-x)\sin x + x\cos x\}$

(c) $\frac{e^x}{2}\{(1+x)\sin x - x\cos x\}$

(d) none of these

3. If $I = \int \frac{dx}{e^x + 4e^{-x}} = f(x) + c$, then $f(x)$ equals to

(a) $2\tan^{-1}(2e^x)$ (b) $\frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right)$

(c) $2\tan^{-1}\frac{e^x}{2}$ (d) $\frac{1}{2}\tan^{-1}(2e^{2x})$

4. The value of $I = \int \frac{\sin x}{\sin x - \cos x}dx$ is

(a) $\frac{x}{2} - \frac{1}{2}\log(\sin x - \cos x) + c$

(b) $\frac{x}{2} + \frac{1}{2}\log(\sin x - \cos x) + c$

(c) $\frac{x}{2} + \frac{1}{2}\log(\sin x + \cos x) + c$

(d) none of these

5. $\int \frac{(1+x)}{x(1+xe^x)^2}dx$ is equal to

(a) $\ln\left(\frac{xe^x}{1+xe^x}\right) - \left(\frac{1}{1+xe^x}\right) + c$

(b) $\ln\left(\dfrac{xe^x}{1+xe^x}\right)+\left(\dfrac{1}{1-xe^x}\right)+c$

(c) $\ln\left(\dfrac{xe^x}{1-xe^x}\right)+\left(\dfrac{1}{1+xe^x}\right)+c$

(d) $\ln\left(\dfrac{xe^x}{1+xe^x}\right)+\left(\dfrac{1}{1+xe^x}\right)+c$

6. $\displaystyle\int\frac{\cos 5x+\cos 4x}{1-2\cos 3x}dx$ is equal to

(a) $\dfrac{\sin 2x}{2}-\sin x+c$ (b) $-\dfrac{\sin 2x}{2}+\sin x+c$

(c) $-\dfrac{\sin 2x}{2}-\sin x+c$ (d) $\dfrac{\sin 2x}{2}+\sin x+c$

7. $\displaystyle\int\frac{\sec x\ \mathrm{cosec}\ x}{\log\tan x}dx$ is equal to

(a) $\log(\tan x)+c$ (b) $\cot(\log x)+c$

(c) $\log\log(\tan x)+c$ (d) $\tan(\log x)+c$

8. $\displaystyle\int\frac{\left(x^4-x\right)^{\frac{1}{4}}}{x^5}dx$ is equal to

(a) $\dfrac{4}{15}\left(1-\dfrac{1}{x^3}\right)^{\frac{5}{4}}+c$ (b) $\dfrac{4}{5}\left(1-\dfrac{1}{x^3}\right)^{\frac{5}{4}}+c$

(c) $\dfrac{4}{15}\left(1+\dfrac{1}{x^3}\right)^{\frac{5}{4}}+c$ (d) none of these

9. $\displaystyle\int\frac{\log(x+1)-\log x}{x(x+1)}dx$ equals

(a) $-\log\left(\dfrac{x+1}{x}\right)+c$ (b) $-\log\left[\log\left(\dfrac{x+1}{x}\right)\right]+c$

(c) $-\dfrac{1}{2}\left[\log\left(\dfrac{x+1}{x}\right)\right]^2+c$

(d) $c-\dfrac{1}{2}[\log(x+1)^2-(\log x)^2]$

10. $\displaystyle\int\frac{\sin x-\cos x}{\sqrt{1-\sin 2x}}e^{\sin x}\cos x\,dx$ is equal to

(a) $e^{\sin x}+c$ (b) $e^{\sin x-\cos x}+c$

(c) $e^{\sin x+\cos x}+c$ (d) $e^{\cos x-\sin x}+c$

11. The value of $I=\displaystyle\int\frac{(1-\cos x)\,dx}{\cos x(1+\cos x)}$ is

(a) $\log(\sec x+\tan x)-2\tan\left(\dfrac{x}{2}\right)+c$

(b) $\log(\sec x+\tan x)+2\tan\left(\dfrac{x}{2}\right)+c$

(c) $\log(\sec x-\tan x)-2\tan\left(\dfrac{x}{2}\right)+c$

(d) none of these

12. $\displaystyle\int\{\sin(\log x)+\cos(\log x)\}dx$ is equal to

(a) $x\sin(\log x)+c$ (b) $x\cos(\log x)+c$

(c) $x\log(\sin x)+c$ (d) $x\log(\cos x)+c$

13. $\displaystyle\int\frac{dx}{\cos^6 x+\sin^6 x}$ is equal to

(a) $\log_e(\tan x-\cot x)+c$

(b) $\log_e(\cot x-\tan x)+c$

(c) $\tan^{-1}(\tan x-\cot x)+c$

(d) $2\tan^{-1}(2\cot 2x)+c$

14. $\displaystyle\int\frac{3+2\cos x}{(2+3\cos x)^2}dx$ is equal to

(a) $\left(\dfrac{\sin x}{3\cos x+2}\right)+c$ (b) $\left(\dfrac{2\cos x}{3\sin x+2}\right)+c$

(c) $\left(\dfrac{2\cos x}{3\cos x+2}\right)+c$ (d) $\left(\dfrac{2\sin x}{3\sin x+2}\right)+c$

15. $\displaystyle\int\sin(\ln x)\,dx$ is equal to

(a) $\dfrac{x}{2}[\sin(\ln x)+\cos(\ln x)]+c$

(b) $\dfrac{x}{2}[\cos(\ln x)-\sin(\ln x)]+c$

(c) $\dfrac{x}{2}[\sin(\ln x)-\cos(\ln x)]+c$

(d) $x[\sin(\ln x)-\cos(\ln x)]+c$

16. $\displaystyle\int\frac{1}{x[(\ln x)^2+4\ln x-1]}dx$ is equal to

(a) $\dfrac{1}{\sqrt{5}}\ln\left|\dfrac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$ (b) $\dfrac{1}{2}\ln\left|\dfrac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$

(c) $\dfrac{1}{2\sqrt{5}}\ln\left|\dfrac{\ln x+2-\sqrt{5}}{\ln x-2+\sqrt{5}}\right|+c$

(d) $\dfrac{1}{2\sqrt{5}}\ln\left|\dfrac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$

17. $\displaystyle\int\frac{\sin x+\cos x}{9+16\sin 2x}dx$ is equal to

(a) $\dfrac{1}{40}\ln\left|\dfrac{5+4(\sin x-\cos x)}{5-4(\sin x-\cos x)}\right|+c$

(b) $\frac{1}{40}\ln\left|\frac{5-4(\sin x-\cos x)}{5+4(\sin x-\cos x)}\right|+c$

(c) $\frac{1}{40}\ln\left|\frac{5+4(\sin x+\cos x)}{5-4(\sin x+\cos x)}\right|+c$

(d) $\frac{1}{40}\ln\left|\frac{5-4(\sin x+\cos x)}{5+4(\sin x-\cos x)}\right|+c$

18. For what values of a and b, the equation $\int(\sin 2x-\cos 2x)dx=\frac{1}{\sqrt{2}}\sin(2x-a)+b$ holds good?

(a) $a=-\frac{5\pi}{4}$, b is any arbitrary constant

(b) $a=\frac{5\pi}{4}$, b is any arbitrary constant

(c) $a=-\frac{\pi}{4}$, b is any arbitrary constant

(d) $a=\frac{\pi}{4}$, b is any arbitrary constant

19. $\int\cos\left(\ln\frac{x}{a}\right)dx$ is equal to

(a) $x\left[\cos\left(\ln\frac{x}{a}\right)-\sin\left(\ln\frac{x}{a}\right)\right]+c$

(b) $\frac{x}{2}\left[\cos\left(\ln\frac{x}{a}\right)+\sin\left(\ln\frac{x}{a}\right)\right]+c$

(c) $\frac{x}{2}\left[\cos\left(\ln\frac{x}{a}\right)-\sin\left(\ln\frac{x}{a}\right)\right]+c$

(d) $x\left[\cos\left(\ln\frac{x}{a}\right)+\sin\left(\ln\frac{x}{a}\right)\right]+c$

20. $\int\frac{x\cos x}{(x\sin x+\cos x)^2}dx$ is equal to

(a) $\frac{1}{(x\sin x+\cos x)^2}+c$ (b) $-\frac{1}{(x\sin x+\cos x)}+c$

(c) $\frac{1}{(x\sin x+\cos x)^3}+c$ (d) $\frac{1}{(x\sin x+\cos x)^4}+c$

21. $\int\frac{(\sqrt{x^2+1})\{\ln(x^2+1)-2\ln x\}}{x^4}dx$ is equal to

(a) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(b) $\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(c) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(d) $\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

22. $\int\frac{dx}{5+4\cos x}$ is equal to

(a) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$ (b) $-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$

(c) $\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$ (d) none of these

23. If $I_n=\int(\ln x)^n dx$, then I_n+nI_{n-1} is equal to

(a) $x(\ln x)^{n-1}$ (b) $x(\ln x)^n$

(c) $nx(\ln x)^n$ (d) none of these

Multiple Correct Answer Type

24. Let $G(x,t)=\begin{cases}x(t-1), & \text{where } x\le t\\ t(x-1), & \text{where } t<x\end{cases}$ and t is continuous function of x in [0, 1].

If $g(x)=\int_0^1 f(t)G(x,t)dt$, then

(a) $g(0)=1$ (b) $g(0)=0$

(c) $g(1)=1$ (d) $g''(x)=f(x)$

25. Let $T>0$ be a fixed real number. Suppose $f(x)$ is a continuous function for all $x\in R$, $f(x+T)=f(x)$. If $I=\int_0^T f(x)dx$, then

(a) $\int_5^{5+5T}f(x)dx=5I$ (b) $\int_5^{5+5T}f(2x)dx=10I$

(c) $\int_5^{5+5T}f(3x)dx=5I$ (d) $\int_5^{5+5T}f(3x)dx=15I$

26. If $I_1=\int_0^1 2^{x^2}dx$, $I_2=\int_0^1 2^{x^3}dx$, $I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$, then

(a) $I_1>I_2$ (b) $I_2>I_1$ (c) $I_3>I_4$ (d) $I_4>I_3$

27. The value of $\int_{1/e}^{\tan x}\frac{tdt}{1+t^2}+\int_{1/e}^{\cot x}\frac{dt}{t(1+t^2)}$ is equal to

(a) $\frac{1}{2+\tan^2 x}$ (b) 1

(c) $\frac{\pi}{4}$ (d) $\frac{2}{\pi}\int_{-1}^{1}\frac{dt}{1+t^2}$

28. If $J = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2}dx$, then J equals to

(a) $\frac{x^7}{2x^7 + x^2 + 1} + c$ (b) $\frac{x^5}{x^2 + 1 + 2x^7} + c$

(c) $\frac{-1}{2x^7 + x^2 + 1} + c$

(d) $\frac{p(x)}{q(x)} + c, \deg(p(x)) = \deg(q(x)) = 7$

Comprehension Type

Paragraph for Question No. 29 & 30

Given: $\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = \alpha(b)f(b) - \alpha(a)f(a)$

29. $\int_0^3 (x^2 + 1)d[x]$ (where $[\cdot]$ denotes the greatest integer function) is equal to

(a) 3 (b) $\frac{9}{2}$ (c) 17 (d) $\frac{27}{2}$

30. $\int_0^6 (x^2 + [x])d|3 - x|$ (where $[\cdot]$ denotes the greatest integer function) is equal to

(a) 36 (b) 72 (c) 63 (d) 126

Paragraph for Question No. 31 to 33

Let n be a non-negative integer and, let $I_n = \int x^n\sqrt{a^2 - x^2}\,dx\,(a > 0)$ we can find relation among I_n, I_{n-1}, I_{n-2}. It can be observed that I_1 is elementary integration whose value is $-\frac{1}{3}(a^2 - x^2)^{3/2}$.

If $I_n = -\frac{x^{n-1}(a^2 - x^2)^{3/2}}{A} + a^2BI_{n-2}$, where A and B are constants. Then

31. A must be equal to

(a) $n + 1$ (b) $n - 1$ (c) $n + 2$ (d) n

32. B must be equal to

(a) $\frac{n+1}{n+2}$ (b) $\frac{n+2}{n+1}$ (c) $\frac{n}{n+2}$ (d) $\frac{n-1}{n+2}$

33. The value of $\int_0^a x^4\sqrt{a^2 - x^2}\,dx$ must be

(a) $\frac{\pi a^6}{32}$ (b) $\frac{\pi a^4}{16}$ (c) $\frac{\pi a^4}{64}$ (d) $\frac{\pi a^6}{16}$

Matrix-Match Type

34. Match the following.

Column-I		Column-II	
A.	$\int\sqrt{\operatorname{cosec} x + 1}\,dx =$	p.	$\frac{1}{\sqrt{2}}\cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) + c$
B.	$\int\frac{\sin x}{\sqrt{3 - \cos 2x}}dx =$	q.	$\sin^{-1}(\sin x\cos x) + c$
C.	$\int\frac{\cos x}{\sqrt{\cos 2x}}dx =$	r.	$\frac{-1}{\sqrt{2}}\sin^{-1}(\sqrt{\cos 2x}) + c$
D.	$\int\frac{\cos 2x}{\sqrt{\cos^2 x + \sin^4 x}}dx =$	s.	$\sin^{-1}(2\sin x - 1) + c$

35. Match the anti-derivative given in column-II of the functions given in column-I.

Column-I		Column-II	
A.	$\frac{\sec x}{(\sec x + \tan x)^2}$	p.	$\log\left\lvert\frac{\sin x - 2}{\sin x - 1}\right\rvert + c$
B.	$\frac{\cos x}{(\sin x - 1)(\sin x - 2)}$	q.	$-\frac{\cos^2 x}{2(1 + \sin x)^2} + c$
C.	$\sin^{-1}\left(\frac{2x}{1 + x^2}\right), \lvert x\rvert < 1$	r.	$2x\tan^{-1}x - \log(1 + x^2) + c$
D.	$\sqrt{\tan x} + \sqrt{\cot x}$	s.	$\sqrt{2}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + c$

mtG
30 DAYS JEE MAIN
Revision Cum Crash Course
₹ 350 ₹ 350 ₹ 350
MTG Learning Media (P) Ltd.
Plot 99, Sector 44, Institutional Area, Gurugram - 122 003
Telefax: (0124) 6601200 e-mail : info@mtg.in
Buy ONLINE at
www.mtg.in

SOLUTIONS

1. (a) : Putting $x = a\cos^2\theta + b\sin^2\theta$ in the given integral, we get

$$\int \frac{2(b-a)\sin\theta\cos\theta d\theta}{\sqrt{(a\cos^2\theta + b\sin^2\theta - a)(b - a\cos^2\theta - b\sin^2\theta)}}$$

$$= \int \frac{2(b-a)\sin\theta\cos\theta d\theta}{(b-a)\sin\theta\cos\theta} = \int 2\ d\theta$$

$$= 2\theta + c = 2\sin^{-1}\sqrt{\left(\frac{x-a}{b-a}\right)} + c$$

2. (a) : Let $I = \text{real part of} \int x e^{(1+i)x} dx$

Now, $\int xe^{(1+i)x}\ dx = \frac{xe^{(1+i)x}}{1+i} - \int \frac{e^{(1+i)x}}{1+i} dx$

$$= \frac{xe^{(1+i)x}}{1+i} - \frac{e^{(1+i)x}}{(1+i)^2} = e^{(1+i)x}\left[\frac{x(1+i)-1}{(1+i)^2}\right]$$

$$= e^x(\cos x + i\sin x)\left[\frac{(x-1)+ix}{1+2i-1}\right]$$

$$= \frac{e^x}{-2}[i\cos x - \sin x][(x-1)+ix]$$

$$\therefore\quad I = -\frac{e^x}{2}[(1-x)\sin x - x\cos x] + c$$

3. (b)

4. (b) : Let $\sin x = A(\sin x - \cos x) + B(\cos x + \sin x)$

$\Rightarrow\quad \sin x = (A + B)\sin x + (B - A)\cos x$

$\Rightarrow\quad A + B = 1$ and $B - A = 0 \Rightarrow A = B = 1/2$

$$\therefore\quad I = \int \frac{\frac{1}{2}(\sin x - \cos x) + \frac{1}{2}(\cos x + \sin x)}{\sin x - \cos x} dx$$

$$= \frac{x}{2} + \frac{1}{2}\log(\sin x - \cos x) + c$$

5. (d) : Let $I = \int \frac{(1+x)}{x(1+xe^x)^2} dx = \int \frac{(1+x)e^x}{(xe^x)(1+xe^x)^2} dx$

Put $1 + xe^x = p \Rightarrow e^x(1+x)dx = dp \ \therefore\ I = \int \frac{dp}{(p-1)p^2}$

Now let, $\frac{1}{(p-1)p^2} = \frac{A}{(p-1)} + \frac{B}{p} + \frac{C}{p^2}$

$\Rightarrow\quad 1 = Ap^2 + Bp(p - 1) + C(p - 1)$

For $p = 1$, $p = 0$ and $p = -1$, we get

$A = 1$, $C = -1$ and $B = -1$

$$\therefore\quad I = \int \frac{1}{(p-1)} dp - \int \frac{dp}{p} - \int \frac{dp}{p^2} = \ln\frac{(p-1)}{p} + \frac{1}{p} + c$$

$$= \ln\left(\frac{xe^x}{1+xe^x}\right) + \left(\frac{1}{1+xe^x}\right) + c$$

6. (c) : We have, $\int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x} dx$

$$= \int \frac{2\cos\frac{9x}{2}\cos\frac{x}{2}\cos\frac{3x}{2}}{\left[1 - 2\left(2\cos^2\frac{3x}{2} - 1\right)\right]\cos\frac{3x}{2}} dx$$

$$= \int \frac{2\cos\frac{9x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{3\cos\frac{3x}{2} - 4\cos^3\frac{3x}{2}} dx = \int \frac{2\cos\frac{9x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{-\cos\frac{9x}{2}} dx$$

$$= -\int(\cos 2x + \cos x)\,dx = -\frac{\sin 2x}{2} - \sin x + c$$

7. (c) **8. (a)** **9. (c)** **10. (a)**

11. (a) : Given, $I = \int \frac{(1-\cos x)\,dx}{\cos x(1+\cos x)}$

$$= \int\left(\frac{1}{\cos x} - \frac{2}{1+\cos x}\right)dx = \int\left(\sec x - \sec^2\frac{x}{2}\right)dx$$

$$= \log(\sec x + \tan x) - 2\tan\left(\frac{x}{2}\right) + c$$

12. (a)

13. (c) : Let $I = \int \frac{dx}{\cos^6 x + \sin^6 x} = \int \frac{\sec^6 x}{1 + \tan^6 x} dx$

$$= \int \frac{(1+\tan^2 x)^2\sec^2 x dx}{1 + \tan^6 x}$$

Put $\tan x = p \Rightarrow \sec^2 x\ dx = dp$

$$\therefore\quad I = \int \frac{(1+p^2)^2 dp}{1+p^6} = \int \frac{(1+p^2)}{p^4 - p^2 + 1} dp$$

$$= \int \frac{p^2\left(1 + \frac{1}{p^2}\right)}{p^2\left(p^2 + \frac{1}{p^2} - 1\right)} dp$$

Put $p - \frac{1}{p} = k \Rightarrow \left(1 + \frac{1}{p^2}\right)dp = dk$

$$\therefore\quad I = \int \frac{dk}{k^2 + 1} = \tan^{-1}(k) + c$$

$= \tan^{-1}\left(p - \frac{1}{p}\right) + c = \tan^{-1}(\tan x - \cot x) + c$

14. (a) : Let $I=\int \frac{3+2\cos x}{(2+3\cos x)^2}dx$

$$=\int \frac{(3\operatorname{cosec}^2 x+2\cot x\ \operatorname{cosec} x)}{(2\operatorname{cosec} x\ +\ 3\cot\ x)^2}dx$$

$$=-\int \frac{-3\operatorname{cosec}^2 x-2\cot x\ \operatorname{cosec} x}{(2\operatorname{cosec} x+3\cot x)^2}dx$$

$$=\frac{1}{2\operatorname{cosec} x+3\cot x}+c=\left(\frac{\sin x}{2+3\cos x}\right)+c$$

15. (c)

16. (d) : Let $I=\int \frac{1}{x[(\ln x)^2+4\ln x-1]}dx$

Put ln x = t ⇒ $\frac{1}{x}dx=dt$

$$\therefore\quad I=\int\frac{dt}{t^2+4t-1}=\int\frac{dt}{(t+2)^2-(\sqrt{5})^2}$$

$$=\frac{1}{2\sqrt{5}}\ln\left|\frac{t+2-\sqrt{5}}{t+2+\sqrt{5}}\right|+c=\frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$$

17. (a) : Let $I=\int\frac{\sin x+\cos x}{9+16\sin 2x}dx$

Put t = sin$x$ − cos$x$ ⇒ dt = (sin x + cos x)dx

Also, t^2 = 1 − sin2x ⇒ sin2x = 1 − t^2

$$\therefore\quad I=\int\frac{dt}{9+16(1-t^2)}=\int\frac{dt}{25-16t^2}$$

$$=\frac{1}{16}\int\frac{dt}{\left(\frac{5}{4}\right)^2-t^2}=\frac{1}{16}\times\frac{1}{2\left(\frac{5}{4}\right)}\ln\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right|+c$$

$$=\frac{1}{40}\ln\left|\frac{5+4t}{5-4t}\right|+c=\frac{1}{40}\ln\left|\frac{5+4(\sin x-\cos x)}{5-4(\sin x-\cos x)}\right|+c$$

18. (a) : We have, $\int(\sin 2x-\cos 2x)\,dx$

$$=-\int\sqrt{2}\ \cos\left(2x+\frac{\pi}{4}\right)dx$$

$$=\frac{-\sqrt{2}}{2}\sin\left(2x+\frac{\pi}{4}\right)+c=\frac{1}{\sqrt{2}}\sin\left(2x+\frac{5\pi}{4}\right)+c$$

$\therefore\ a=-\frac{5\pi}{4}$, b is any arbitrary constant.

19. (b) **20. (b)**

21. (d) : Let $I=\int\frac{\left(\sqrt{\frac{x^2+1}{x^2}}\right)\ln\left(\frac{x^2+1}{x^2}\right)}{x^3}dx$

Put $\frac{x^2+1}{x^2}=t\ \Rightarrow\ -\frac{2}{x^3}dx=dt$

$$\therefore\quad I=-\frac{1}{2}\int\sqrt{t}\ln t\,dt\ =-\frac{1}{2}\left[(\ln t).\frac{2t^{3/2}}{3}-\frac{2}{3}\int\frac{1}{t}.t^{3/2}dt\right]$$

$$=\frac{1}{9}t^{3/2}\,[2-3\ln t]+c$$

$$=\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$$

22. (a) **23. (b)**

24. (b, d) : For x = 0, G(0, t) = 0, t ≥ 0

$$g(0)=\int_0^1 f(t)\,G\,(0,t)\,dt=0$$

Also, for x = 1, G(1, t) = 0, t < 1

$$g(1)=\int_0^1 f(t)\,G\,(1,t)\,dt=0$$

Now, $g(x)=\int_0^x f(t)\,G\,(x,t)\,dt+\int_x^1 f(t)\,G\,(x,t)\,dt$

MtG
ONLINE TEST SERIES
Practice Part Syllabus/ Full Syllabus
24 Mock Tests for
JEE Main
Now on your android smart phones
with the same login of web portal.
Log on to test.pcmbtoday.com

$$= (x-1)\int_0^x tf(t)dt + x\int_x^1 (t-1)\,f(t)\,dt$$

$$\Rightarrow g'(x) = \int_0^x tf(t)dt + \int_x^1 (t-1)\,f(t)\,dt$$

$$\therefore\ g''(x) = xf(x) + \{0-(x-1)f(x)\} = f(x)$$

25. (a, c) : $\int_a^{a+nT} f(x)dx = n\int_0^T f(x)dx$

26. (a, d) : For $0 < x < 1$. We know that $x^2 > x^3$.
For $1 < x < 2$, we have $x^3 > x^2$.
Thus, $2^{x^2} > 2^{x^3}$ for $x \in (0, 1)$ and $2^{x^3} > 2^{x^2}$ for $x \in (1, 2)$
So, $I_1 > I_2$ and $I_4 > I_3$

27. (b, d) : Let $I = \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$

Put $t = \frac{1}{z} \Rightarrow dt = -\frac{1}{z^2}dz$

$$\therefore\ I = \int_e^{\tan x} \frac{-\frac{1}{z^2}dz}{\frac{1}{z}\left(1+\frac{1}{z^2}\right)} = \int_{\tan x}^{e} \frac{zdz}{(z^2+1)} = \int_{\tan x}^{e} \frac{tdt}{(1+t^2)}$$

$$\therefore\ \int_{1/e}^{\tan x} \frac{tdt}{(1+t^2)} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$$

$$= \int_{1/e}^{\tan x} \frac{t}{(1+t^2)}dt + \int_{\tan x}^{e} \frac{t}{(1+t^2)}dt$$

$$= \int_{1/e}^{e} \frac{tdt}{(1+t^2)} = \frac{1}{2}[\ln(1+t^2)]_{1/e}^{e}$$

$$= \frac{1}{2}\left\{\ln(1+e^2) - \ln\left(1+\frac{1}{e^2}\right)\right\} = \frac{1}{2}(\ln e^2) = 1$$

Also, $\frac{2}{\pi}\int_{-1}^{1} \frac{dt}{1+t^2} = \frac{4}{\pi}\int_0^1 \frac{dt}{1+t^2} = \frac{4}{\pi}\cdot\tan^{-1}1 = \frac{4}{\pi}\cdot\frac{\pi}{4} = 1$

28. (a, d)

29. (c) : Here, $f(x) = x^2 + 1$, $\alpha(x) = [x]$

$$\therefore\ \int_0^3 (x^2+1)d[x] + \int_0^3 [x]2xdx = [3](9+1) - 0$$

$$\Rightarrow\ \int_0^3 (x^2+1)d[x] = 30 - 2\int_0^3 x[x]dx$$

$$= 30 - 2\left\{\int_0^1 0\cdot dx + \int_1^2 xdx + \int_2^3 2xdx\right\}$$

$$= 30 - 2\left\{0 + \frac{3}{2} + 5\right\} = 30 - 3 - 10 = 17$$

30. (c) : $\int_0^6 (x^2 + [x])\,d\,|3-x|$

$$= \int_0^3 (x^2+[x])d(3-x) + \int_3^6 (x^2+[x])d(x-3)$$

$$= -\int_0^3 x^2dx + \int_3^6 x^2dx - \int_0^3 [x]dx + \int_3^6 [x]dx$$

$$= -\left[\frac{x^3}{3}\right]_0^3 + \left[\frac{x^3}{3}\right]_3^6 - \int_0^3 [x]dx + \int_0^3 [x+3]dx$$

$$= -9 + (72-9) - \int_0^3 [x]\,dx + \int_0^3 [x]\,dx + 3\int_0^3 1\,dx = 72 - 9 = 63$$

(31-32) : 31. (c) **32. (d)**

$$I_n = \int x^n\sqrt{a^2-x^2}\,dx = \int x^{n-1}(x\sqrt{a^2-x^2})dx$$

Applying integration by parts, we get

$$I_n = -\frac{1}{3}(a^2-x^2)^{3/2}x^{n-1} + \frac{n-1}{3}\int x^{n-2}(a^2-x^2)^{3/2}dx$$

$$= -\frac{1}{3}(a^2-x^2)^{3/2}x^{n-1} + \frac{n-1}{3}\int (a^2x^{n-2}\sqrt{a^2-x^2} - x^n\sqrt{a^2-x^2})dx$$

$$\Rightarrow\ I_n = -\frac{1}{3}(a^2-x^2)^{3/2}x^{n-1} + \frac{n-1}{3}a^2I_{n-2} - \frac{n-1}{3}I_n$$

$$\Rightarrow\ I_n = \frac{-(a^2-x^2)^{3/2}x^{n-1}}{n+2} + \left(\frac{n-1}{n+2}\right)a^2I_{n-2}$$

$$\therefore\ A = n+2 \text{ and } B = \frac{n-1}{n+2}$$

33. (a) : If $I_n = \int_0^a x^n\sqrt{a^2-x^2}\,dx \Rightarrow I_n = \left(\frac{n-1}{n+2}\right)a^2I_{n-2}$

$$\therefore\ I_4 = \frac{3}{6}a^2I_2,\ I_2 = \frac{1}{4}a^2I_0 \text{ and } I_0 = \frac{\pi a^2}{4}$$

$$\Rightarrow\ I_4 = \frac{3}{6}\times\frac{1}{4}\times\frac{\pi}{4}a^6 = \frac{\pi a^6}{32}$$

34. A → s; B → p; C → r; D → q

(A) $\int \sqrt{\operatorname{cosec} x+1}\,dx=\int \frac{\sqrt{1+\sin x}}{\sqrt{\sin x}}\,dx$

$=\int \frac{\cos x}{\sqrt{\sin x(1-\sin x)}}dx=\int \frac{dt}{\sqrt{t(1-t)}}$

$= \sin^{-1}(2\sin x - 1) + c$

(B) $\int \frac{\sin x}{\sqrt{3-\cos 2x}}\,dx=\frac{1}{\sqrt{2}}\int \frac{\sin x}{\sqrt{2-\cos^2 x}}\,dx$

$=\frac{-1}{\sqrt{2}}\int \frac{dt}{\sqrt{2-t^2}}=\frac{1}{\sqrt{2}}\cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)+c$

(C) $\int \frac{\cos x}{\sqrt{\cos 2x}}\,dx=\frac{1}{\sqrt{2}}\int \frac{\sin 2x}{\sqrt{1-\cos 2x}\sqrt{\cos 2x}}\,dx$

$=-\frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{1-t^2}}\,dt=-\frac{1}{\sqrt{2}}\sin^{-1}(\sqrt{\cos 2x})+c$

(D) $\int \frac{\cos 2x}{\sqrt{\cos^2 x+\sin^4 x}}\,dx=\int \frac{2\cos 2x}{\sqrt{4-\sin^2 2x}}\,dx$

$=\int \frac{dt}{\sqrt{4-t^2}}=\sin^{-1}\left(\frac{\sin 2x}{2}\right)+c$

35. A → q ; B → p ; C → r ; D → s

(A) $\int \frac{\sec x}{(\sec x+\tan x)^2}\,dx=\int \frac{\sec x(\sec x+\tan x)}{(\sec x+\tan x)^3}\,dx$

$=\int \frac{dt}{t^3}=-\frac{1}{2}(\sec x+\tan x)^{-2}+c=\frac{-\cos^2 x}{2(1+\sin x)^2}+c$

(B) $\int \frac{\cos x}{(\sin x-1)(\sin x-2)}\,dx=\int \frac{dt}{(t-1)(t-2)}$

$=\int\left(\frac{1}{t-2}-\frac{1}{t-1}\right)dt=\log\left|\frac{t-2}{t-1}\right|+c=\log\left|\frac{\sin x-2}{\sin x-1}\right|+c$

(C) $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx=2\int t\sec^2 t\,dt$

$= 2[t \tan t - \log|\sec t|] + c$

$= 2x \tan^{-1}x - \log(1 + x^2) + c$

(D) $\int(\sqrt{\tan x}+\sqrt{\cot x})dx=\int\left(y+\frac{1}{y}\right)\frac{2y}{1+y^4}\,dy$

[By putting $\tan x = y^2 \Rightarrow dx = 2y\,dy/(1 + y^4)$]

$=2\int \frac{y^2+1}{1+y^4}\,dy=2\int \frac{1+1/y^2}{y^2+1/y^2}\,dy$

$=2\int \frac{1+1/y^2}{(y-1/y)^2+2}\,dy=2\int \frac{du}{u^2+2}$

$\left[\text{By putting } u=y-1/y \Rightarrow du=\left(1+\frac{1}{y^2}\right)dy\right]$

$=2\frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}}+c=\sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+c$

◆ ◆

The Ministry of Education has launched a new programme to train over 5 lakh students across India in cyber skills. The initiative is a collaborative effort of AICTE, MoE, and CyberPeace Foundation, a think tank and an NGO. The programme aims to address issues that have emerged in the wake of technological advancements, including cybercrimes, harassment, abuse, and radicalisation. The govt claims this would help in building "a secure ecosystem of cybersecurity communities where the internet will become a strength instead of a liability or weakness for the youth of the country".

As a part of the program, 50 CyberPeace clubs will engage five lakhs students and educators in CyberPeace and cybersecurity activities including hackathons, global CyberPeace challenge, and eRaksha competitions. Under the initiative, a network of 25, 000 CyberPeace Corps volunteers will also be formed. Students will be taught subjects including digital citizenship, digital rights and responsibilities, cyber ethics and digital etiquette, cyber health and wellness, misinformation, cyber laws and reporting, and new technologies such as block chain, IoT security and AI. Participants will also be made aware of their digital rights and responsibilities, choose consciously by making safe, smart, informed choices, become hygiene conscious by following healthy online practices, recognize and manage online risk and to become responsible netizens following the rules and regulations, the official statement claims.

"I hereby announce the launch of the training program in Cyber Security through CyberPeace Foundation which is going to train more than five lakhs students and this is very important area where more than two Crore jobs exists," said Anil Sahasrabudhe, AICTE chairperson.

Courtesy: The Indian Express

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 9 : Vector Algebra and Three Dimensional Geometry

Time Taken : 60 Min.

Only One Option Correct Type

1. A unit tangent vector at $t = 2$ on the curve $x = t^2 + 2, y = 4t - 5, z = 2t^2 - 6t$ is

(a) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (b) $\frac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$

(c) $\frac{1}{\sqrt{6}}(2\hat{i} + \hat{j} + \hat{k})$ (d) none of these

2. The projection of a line segment on the coordinates axes are 2, 3, 6. Then, the length of the line segment is

(a) 7 (b) 5 (c) 1 (d) 11

3. The angle between a line with direction ratios proportional to 2, 2, 1 and a line joining (3, 1, 4) to (7, 2, 12), is

(a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\cos^{-1}\left(-\frac{2}{3}\right)$

(c) $\tan^{-1}\left(\frac{2}{3}\right)$ (d) none of these

4. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{d}$ be a non-zero vector which is perpendicular to $\vec{a} + \vec{b} + \vec{c}$. If $\vec{d} = x\vec{b} \times \vec{c} + y\vec{c} \times \vec{a} + z\vec{a} \times \vec{b}$, then

(a) $xy + yz + zx = 0$ (b) $x = y = z$

(c) $x^3 + y^3 + z^3 = 3xyz$ (d) $x + y + z = 1$

5. The line passing through the points $(5, 1, a)$ and $(3, b, 1)$ crosses yz plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

(a) $a = 2, b = 8$ (b) $a = 4, b = 6$

(c) $a = 6, b = 4$ (d) $a = 8, b = 2$

6. If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to

(a) 16 (b) 8 (c) 3 (d) 12

One or More Than One Option(s) Correct Type

7. Let $A_1, A_2, \ldots, A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point A_k, $k = 1, 2, \ldots, n$. If $\left|\sum_{k=1}^{n-1}(\vec{a}_k \times \vec{a}_{k+1})\right| = \left|\sum_{k=1}^{n-1}(\vec{a}_k \cdot \vec{a}_{k+1})\right|$, then the minimum value of n is

(a) 1 (b) 2 (c) 8 (d) 9

8. Let P_1 denote the equation of the plane to which the vector $(\hat{j} + \hat{i})$ is normal and which contains the line L whose equation is $\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Let P_2 denote the equation of the plane containing the line L and a point with position vectors $\hat{j}$. Which of the following holds goods?

(a) Equation of P_1 is $x + y = 2$

(b) Equation of P_2 is $\vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 2$.

(c) The acute angle between P_1 and P_2 is $\cot^{-1}(\sqrt{3})$.

(d) The angle between the plane P_2 and the line L is $\tan^{-1}\sqrt{3}$.

9. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j}$, then the vectors $(\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}, (\vec{b} \cdot \hat{i})\hat{i} + (\vec{b} \cdot \hat{j})\hat{j} + (\vec{b} \cdot \hat{k})\hat{k}$, and $\hat{i} + \hat{j} - 2\hat{k}$

(a) are mutually perpendicular

(b) are coplanar

(c) form a parallelopiped of volume 6 units

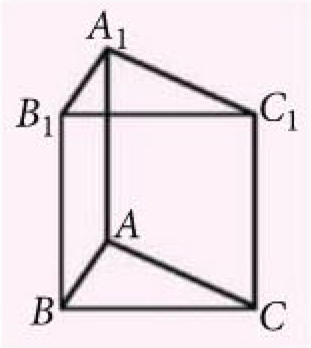
(d) form a parallelopiped of volume 3 units

10. The projection of the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ on the plane $x + 2y + z = 9$ is the line of intersection of which of the following planes?

16331955

(a) $x + 2y + z - 9 = 0$ (b) $9x + 2y + 3z - 4 = 0$
(c) $x - 2y + z + 3 = 0$ (d) $9x - 2y - 5z + 4 = 0$

11. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. Find the co-ordinates of the vertex A_1, if the co-ordinates of the base vertices of the prism are $A(1, 0, 1)$, $B(2, 0, 0)$ and $C(0, 1, 0)$.



(a) $(-2, 0, 2)$ (b) $(0, -2, 0)$
(c) $(0, 2, 0)$ (d) $(2, 2, 2)$

12. The equations of two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle 60° is

(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ (b) $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$
(c) $\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$ (d) $\frac{x}{2} = \frac{y}{-1} = \frac{z}{1}$

13. The line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 0$ and $\vec{r} \cdot (3\hat{i} + 2\hat{j} + \hat{k}) = 0$ is

(a) equally inclined to $\hat{i}$ and $\hat{k}$
(b) equally inclined to $\hat{i}$ and $\hat{j}$
(c) inclined at an angle $\cos^{-1}\sqrt{\frac{2}{3}}$ with $\hat{j}$
(d) inclined at an angle $\cos^{-1}\left(\frac{-1}{\sqrt{6}}\right)$ with $\hat{i}$

Comprehension Type

If $\vec{a}, \vec{b}$ and $\vec{c}$ be any three non-coplanar vectors. Then the system $\vec{a}', \vec{b}'$ and $\vec{c}'$ which satisfies
$\vec{a} \cdot \vec{a}' = \vec{b} \cdot \vec{b}' = \vec{c} \cdot \vec{c}' = 1$
and $\vec{a} \cdot \vec{b}' = \vec{a} \cdot \vec{c}' = \vec{b} \cdot \vec{a}' = \vec{b} \cdot \vec{c}' = \vec{c} \cdot \vec{a}' = \vec{c} \cdot \vec{b}' = 0$
is called the reciprocal system to the vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

14. The value of $(\vec{a} \times \vec{a}') + (\vec{b} \times \vec{b}') + (\vec{c} \times \vec{c}')$

(a) $= \vec{a} + \vec{b} + \vec{c}$ (b) $= \vec{a}' + \vec{b}' + \vec{c}'$
(c) $\neq 0$ (d) $= 0$

15. The value of $[\vec{a}'\ \vec{b}'\ \vec{c}']^{-1}$

(a) $< [\vec{a}\ \vec{b}\ \vec{c}]$ (b) $= [\vec{a}\ \vec{b}\ \vec{c}]$
(c) $> [\vec{a}\ \vec{b}\ \vec{c}]$ (d) $= 0$

Matrix Match Type

16. Match the following columns :

	Column-I	Column-II	
P.	The number of independent constants in the equation of a straight line in 3 dimensions is	**1.**	2
Q.	A plane passes through $(1, -2, 1)$ and is perpendicular to two planes $2x - 2y + z = 0$, $x - y + 2z = 4$. If the distance of $(1, 2, 2)$ from it, is $k\sqrt{2}$ units. Then k equals	**2.**	9
R.	If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ are coplanar, then $2k$ is	**3.**	4

	P	Q	R
(a)	3	1	2
(b)	3	2	1
(c)	1	2	3
(d)	2	1	3

Numerical Value Type

17. The distance of the point $\hat{i} + 2\hat{j} + 3\hat{k}$ from the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 5$ measured parallel to the vectors $2\hat{i} + 3\hat{j} - 6\hat{k}$ must be ______.

18. The distance of the point (3, 0, 5) from the line $x - 2y + 2z - 4 = 0 = x + 3z - 11$ is ______.

19. If the lines $x - y + z = -1 = 3x - y + az + 1$ and $x + 4y - z - 1 = 0 = x + 2y + bz + 1$ are perpendicular, then $3ab + a - b + c = 0$, where c is ______.

20. Let $\vec{p} = 2\hat{i} - \hat{j} + \hat{k}, \vec{q} = \hat{i} + 2\hat{j} - k$ and $\vec{r} = \hat{i} + \hat{j} - 2\hat{k}$. If $\vec{V} = \vec{p} + \lambda\vec{q}$ and projection of $\vec{V}$ on $\vec{r}$ is $\frac{4}{\sqrt{6}}$, then value of λ is equal to ______.

❖❖

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

CLASS-XII

Practice questions for CBSE Exams as per the reduced syllabus and marking scheme issued by CBSE for the academic session 2020-21.

Practice Paper 2021

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

(1) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
(2) Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
(3) Both Part A and Part B have internal choices.

Part-A :

(1) It consists of two sections- I and II.
(2) Section I comprises of 16 very short answer type questions.
(3) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part-B :

(1) It consists of three sections- III, IV and V.
(2) Section III comprises of 10 questions of 2 marks each.
(3) Section IV comprises of 7 questions of 3 marks each.
(4) Section V comprises of 3 questions of 5 marks each.
(5) Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART-A

SECTION - I

All questions are compulsory. In case of internal choices attempt any one.

1. If the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is singular, then find x.

OR

What positive value of x makes the following pair of determinant equal?

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$

2. If $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{QO} + \overrightarrow{OR}$, show that the points P, Q, R are collinear.

3. If $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the direction cosines of a line, then find the value of n.

4. Evaluate : $\int \frac{\sqrt{x}}{\sqrt{x^2 - x}}\, dx$

LATEST 2020-21 PRACTICE PAPERS Included

As per the CBSE **Revised Curriculum**
For the Academic Year **2020-21**

MtG

CBSE CHAMPION Chapterwise -Topicwise Solved Papers

CBSE 10 CHAMPION CHAPTERWISE-TOPICWISE
12
PHYSICS
CHEMISTRY
MATHEMATICS
BIOLOGY
ENGLISH CORE
ECONOMICS
BUSINESS STUDIES
ACCOUNTANCY
PHYSICAL EDUCATION
10
SCIENCE
MATHEMATICS
SOCIAL SCIENCE
ENGLISH

CBSE CHAMPION Chapterwise-Topicwise Solved Papers Series contains topicwise questions and solutions asked over last decade in CBSE-Board examination.

Questions are supported with topicwise graphical analysis of previous years CBSE Board questions as well as comprehensive and lucid theory. The questions in each topic have been arranged in descending order as per the marks category. Questions from Delhi, All India, Foreign and Compartment papers are included. This ensures that all types of questions that are necessary for Board exam preparation have been covered.

Important feature of these books is that the solutions to all the questions have been given according to CBSE marking scheme. CBSE sample paper and practice papers are also supplemented.

Examination papers for Class-10 and 12 Boards are based on a certain pattern. To excel, studying right is therefore more important than studying hard, which is why we created this series.

Visit www.mtg.in for latest offers and to buy online!

Available at all leading book shops throughout India.
For more information or for help in placing your order:
Call 0124-6601200 or email info@mtg.in

OR

Evaluate : $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}dx$

5. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, then find the value of 'a' and 'b'.

OR

Write the element a_{23} of a 3 × 3 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$.

6. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 4\}$. Check whether $f = \{(1,1), (1,2), (2,1), (3, 4)\}$ from A to B is a function or not.

7. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, then find the matrix $A^2 - B$.

8. A random variable X has the following distribution.

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the event $E = \{X \text{ is a prime number}\}$, find $P(E)$.

OR

If A and B are two events such that $P(A|B) = p$, $P(A) = p$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{5}{9}$, then find the value of p.

9. Show that $f(x) = x^2 + 1$ is not one-one.

10. Evaluate : $\int \frac{dx}{\sqrt{x^2 - 3x + 2}}$

11. The probability distribution of a random variable X is given below :

X	2	3	4	5
$P(X)$	$5/k$	$7/k$	$9/k$	$11/k$

Find the value of k.

12. Solve the differential equation $\sin\left(\frac{dy}{dx}\right) = a$.

13. Find the direction cosines of a line, for which $\alpha = \beta$ and $\gamma = 45°$.

OR

If $P = (-2, 3, 6)$, then find the d.c.'s of OP.

14. Let $A = \{1, 2, 3, 4\}$ and R be a relation in A given by $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1)\}$. Then, show that R is reflexive.

15. The cartesian equation of a line is $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$. Find its vector equation.

16. If the angle between $\hat{i} + \hat{k}$ and $\hat{i} + \hat{j} + a\hat{k}$ is $\frac{\pi}{3}$, then find the value of a.

SECTION - II

Both the case study based questions are compulsory. Attempt any 4 sub-parts from each question 17 and 18. Each sub-part carries 1 mark.

17. Suppose your friend is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Also, it is given that when it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time.

Based on the above information, answer the following questions :

(i) If an ordinary year is considered, the probability that it rains on wedding day is

(a) 1/365 (b) 1/73

(c) 1/72 (d) 6/365

(ii) The probability that it does not rain on wedding day is

(a) 1/365 (b) 5/365

(c) 360/365 (d) none of these

(iii) The probability that the weatherman predicts correctly is

(a) 5/10 (b) 7/10

(c) 9/10 (d) 1/10

(iv) The probability that it will rain on the wedding day, if weatherman predict rain for tomorrow, is

(a) 0.111 (b) 0.222

(c) 0.333 (d) 0.444

(v) The probability that it will not rain on the wedding day, if weatherman predict rain for tomorrow, is

(a) 0.889 (b) 0.778

(c) 0.667 (d) 0.556

18. A real estate company is going to build a new apartment complex. The land they have purchased can hold at most 5000 apartments. Also, if they make x apartments, then the maintenance costs for the building, landscaping etc., would be as follows:

Fixed cost = ₹ 4000000

Variable cost = ₹$(140x - 0.04x^2)$

Based on the above information, answer the following questions :

(i) The maintenance cost as a function of x will be

(a) $140x - 0.04x^2$ (b) 4000000

(c) $4000000 + 140x - 0.04x^2$

(d) None of these

(ii) If $C(x)$ denote the maintenance cost function, then maximum value of $C(x)$ occur at x =

(a) 0 (b) 1750

(c) 5000 (d) 2000

(iii) The maximum value of $C(x)$ would be

(a) ₹ 5225000 (b) ₹ 4122500

(c) ₹ 5000000 (d) ₹ 4000000

(iv) The number of apartments, that the complex should have in order to minimize the maintenance costs, is

(a) 5000 (b) 4000

(c) 1750 (d) 3500

(v) If the minimum maintenance cost is attain, then the maintenance cost for each apartment would be

(a) ₹ 740 (b) ₹ 540

(c) ₹ 640 (d) ₹ 840

PART-B

SECTION-III

All questions are compulsory. In case of internal choices attempt any one.

19. Determine the value of 'k' for which the following function is continuous at $x = 3$.

$$f(x)=\begin{cases}\dfrac{(x+3)^2-36}{x-3}, & x\neq 3\\ k, & x=3\end{cases}$$

20. Evaluate : $\displaystyle\int \frac{1}{\sqrt{1-\sin x}}\,dx$

OR

Evaluate : $\displaystyle\int \frac{dx}{1-2\sin x\cos x}$

21. Express $\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$, $x \in \left(\dfrac{-\pi}{2}, \dfrac{3\pi}{2}\right)$ in the simplest form.

OR

Evaluate : $\sin\left[2\cos^{-1}\left(\dfrac{-3}{5}\right)\right]$

22. Find the equation of the tangent to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point (1, 3).

23. Find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a} = \hat{i}+\hat{j}+4\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}+4\hat{k}$.

24. For what value(s) of 'a' the matrix $\begin{bmatrix}1 & a & 2\\ 1 & 2 & 5\\ 2 & 1 & 1\end{bmatrix}$ will not be invertible.

25. The equation of a line is $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

OR

Show that the lines $\dfrac{x+1}{3}=\dfrac{y+3}{5}=\dfrac{z+5}{7}$ and $\dfrac{x-2}{1}=\dfrac{y-4}{3}=\dfrac{z-6}{5}$ intersect. Also, find their point of intersection.

CBSE 10th, 12th practical exams from March 1; board exams to commence on May 4

Putting an end to the wait of lakhs of students, Education Minister Ramesh Pokhriyal Nishank announced the CBSE exam dates. The Central Board of Secondary Education (CBSE) will conduct board examinations for Classes 10 and 12 from May 4 to June 10. The results are expected around July 15. It will be held offline or in pen-and-paper mode even as most of the session has been held online. Just like the 2020 academic session, 2021 board exams too will be held amid strict precautions related to the COVID-19 pandemic. Wearing masks will be a must and social distancing will be maintained.

There also will be more questions testing the subject-wise understanding of the student and not the ability to mug-up, claims CBSE. There will be more case-study based questions wherein a paragraph will be given to students and they will have to answer questions after reading the paragraph. This will assess students on their reading, understanding, interpretation, and answer writing abilities and move away from the rot learning, said Joseph Emmanuel, director, academics at CBSE. Sample papers based on the new pattern have also been released on the CBSE official website.

26. Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

27. Using integration, find the area of the region bounded by the line $2y = 5x + 7$, x-axis and the lines $x = 2$ and $x = 8$.

28. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

SECTION-IV

All questions are compulsory. In case of internal choices attempt any one.

29. Find the area enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$.

30. Let N be the set of all natural numbers and let R be a relation in $N \times N$ defined by $(a, b)\ R(c, d) \Rightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$, show that R is an equivalence relation on $N \times N$.

31. Show that the exponential function a^x is (where $a > 0$) continuous at every point.

OR

Check whether the function

$$f(x) = \begin{cases} \dfrac{(3^{\sin x} - 1)^2}{x \cdot \log(1 + x)}, & \text{for } x \neq 0 \\ 2 \log 3, & \text{for } x = 0 \end{cases}$$

is continuous or not.

32. Given the sum of the perimeter of a square and a circle. Show that sum of their areas is least when the side of the square is equal to the diameter of the circle.

33. Evaluate : $\displaystyle\int_0^{\frac{3}{2}} |\, x \cos \pi x \,|\, dx$

34. Solve the differential equation $\left(\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}} - \dfrac{y}{\sqrt{x}}\right)\dfrac{dx}{dy} = 1, x \neq 0.$

OR

Solve the differential equation $\dfrac{dy}{dx} + y \cot x = 2\cos x.$

35. Find the values of a and b, if the function f defined by $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$

is differentiable at $x = 1$.

SECTION-V

All questions are compulsory. In case of internal choices attempt any one.

36. Solve the following LPP graphically.

Maximize $Z = \dfrac{2x}{25} + \dfrac{y}{10}$

Subject to constraints
$x \geq 2000$, $y \geq 4000$
and $x + y \leq 12000$

OR

Maximise $z = 8x + 9y$ subject to the constraints given below :
$2x + 3y \leq 6$, $3x - 2y \leq 6$, $y \leq 1$; $x, y \geq 0$

37. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ and $f(x) = x^2 - 5x - 14$, find $f(A)$. Hence obtain A^3.

OR

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_3$.

38. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

OR

Find the equation of the plane passing through the point $(3, -3, 1)$ and perpendicular to the line joining the points $(3, 4, -1)$ and $(2, -1, 5)$. Also find the coordinates of foot of perpendicular, the equation of perpendicular line and the length of perpendicular drawn from origin to the plane.

SOLUTIONS

1. Given that matrix A is singular $\Rightarrow |A| = 0$

$$\Rightarrow \begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{vmatrix} = 0$$

$\Rightarrow\ 1(-6 - 2) + 2(-3 - x) + 3(2 - 2x) = 0$

$\Rightarrow\ -8 - 6 - 2x + 6 - 6x = 0$

$\Rightarrow\ -8x - 8 = 0 \Rightarrow x = -1$

OR

We have, $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

$\Rightarrow\ 2x^2 - 15 = 32 - 15 \Rightarrow 2x^2 = 32 \Rightarrow x^2 = 16$

$\Rightarrow\ x = 4$ $\quad [\because x > 0]$

2. We have, $\overrightarrow{PO}+\overrightarrow{OQ}=\overrightarrow{QO}+\overrightarrow{OR}$

$\Rightarrow \quad \overrightarrow{PQ}=\overrightarrow{QR}$ [By triangle law]

Thus, $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are either parallel or collinear. But, Q is a point common to them.

So, $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are collinear.

Hence, points P, Q, R are collinear.

3. Since, $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the direction cosines of a line

$$\therefore \quad \left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2+n^2=1 \Rightarrow n^2=\frac{23}{36} \Rightarrow n=\frac{\pm\sqrt{23}}{6}$$

4. Let $I=\int \frac{\sqrt{x}}{\sqrt{x^2-x}}\,dx$

$$=\int \frac{\sqrt{x}}{\sqrt{x(x-1)}}\,dx=\int \frac{dx}{\sqrt{x-1}}=2\sqrt{x-1}+C$$

OR

Let $I=\int \frac{\cos 2x+2\sin^2 x}{\cos^2 x}\,dx$

$$=\int \frac{\cos^2 x-\sin^2 x+2\sin^2 x}{\cos^2 x}\,dx$$

$$=\int \frac{\cos^2 x+\sin^2 x}{\cos^2 x}\,dx=\int \frac{1}{\cos^2 x}\,dx=\int \sec^2 x\,dx$$

$= \tan x + C$

5. Since, matrix A is skew symmetric matrix.

$\therefore \quad A' = -A$ ...(i)

Now, as $A=\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix} \quad \therefore \quad A'=\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$

From (i), $A + A' = O$

$$\Rightarrow \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}+\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}=O$$

$$\Rightarrow \begin{bmatrix} 0 & 2+a & b-3 \\ a+2 & 0 & 0 \\ b-3 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \quad a+2=0$ and $b-3=0 \Rightarrow a=-2$ and $b=3$

OR

Here, $a_{ij}=\frac{|i-j|}{2}$

$\therefore \quad$ For $i=2$, $j=3$ we have, $a_{23}=\frac{|2-3|}{2}=\frac{|-1|}{2}=\frac{1}{2}$

6. Here, f is not a function from A to B as $f(1)$ is not unique.

7. $A^2=\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}=\begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix}=\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$

$$\therefore \quad A^2-B=\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}-\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} 1-0 & -4-4 \\ 12+1 & 1-7 \end{bmatrix}=\begin{bmatrix} 1 & -8 \\ 13 & -6 \end{bmatrix}$$

8. $P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)$
$= 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

OR

We have, $P(A)=p, P(B)=\frac{1}{3}$ and $P(A\cup B)=\frac{5}{9}$

Now, $P(A|B)=\frac{P(A\cap B)}{P(B)}=p \Rightarrow P(A\cap B)=\frac{p}{3}$

Since, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$$\Rightarrow \quad \frac{5}{9}=p+\frac{1}{3}-\frac{p}{3} \Rightarrow \frac{5-3}{9}=\frac{2p}{3} \Rightarrow p=\frac{2}{9}\times\frac{3}{2}=\frac{1}{3}$$

9. Since $f(x) = f(-x) = x^2 + 1$ for all $x \in R$, therefore, f is not one-one.

10. We have, $\int \frac{dx}{\sqrt{x^2-3x+2}}=\int \frac{dx}{\sqrt{\left(x^2-3x+\frac{9}{4}\right)-\frac{1}{4}}}$

$$=\int \frac{dx}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}=\log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+C$$

11. Clearly, $\Sigma P(X) = 1$

$$\Rightarrow \quad \frac{5}{k}+\frac{7}{k}+\frac{9}{k}+\frac{11}{k}=1 \Rightarrow \frac{32}{k}=1 \Rightarrow k=32$$

12. We have, $\sin\left(\frac{dy}{dx}\right)=a$

$$\Rightarrow \quad \frac{dy}{dx}=\sin^{-1} a \quad \Rightarrow \quad dy=\sin^{-1} a\,dx$$

On integrating both sides, we get

$$\int dy=\int \sin^{-1} a\,dx \quad \Rightarrow \quad y=x\cdot\sin^{-1}a+c$$

13. Since, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow \quad 2\cos^2\alpha+\cos^2 45°=1$ $(\because \alpha=\beta)$

$$\Rightarrow \quad 2\cos^2\alpha=1-\frac{1}{2}=\frac{1}{2} \Rightarrow \cos^2\alpha=\frac{1}{4}$$

$\Rightarrow \quad \cos\alpha=\pm\frac{1}{2}$. So, dc's are $\left(\pm\frac{1}{2},\pm\frac{1}{2},\frac{1}{2}\right)$.

OR

Here, $O \equiv (0, 0, 0)$ and $P \equiv (-2, 3, 6)$
Direction ratios of OP are $-2 - 0, 3 - 0, 6 - 0$ *i.e.*, $-2, 3, 6$
∴ Direction cosines of OP are

$$< \frac{-2}{\sqrt{(-2)^2+3^2+6^2}}, \frac{3}{\sqrt{(-2)^2+3^2+6^2}}, \frac{6}{\sqrt{(-2)^2+3^2+6^2}} >$$

i.e., $< \frac{-2}{7}, \frac{3}{7}, \frac{6}{7} >$

14. Here, $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1)\}$
Now, $(1, 1), (2, 2), (3, 3), (4, 4) \in R$
∴ R is reflexive.

15. The given cartesian equation is

$$\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$$

The line passes through the point $(-3, 5, -6)$ and is parallel to vector $2\hat{i} + 4\hat{j} + 2\hat{k}$.
Hence, the vector equation of the line is

$$\vec{r} = -3\hat{i} + 5\hat{j} - 6\hat{k} + \lambda(2\hat{i} + 4\hat{j} + 2\hat{k}).$$

16. We have, $\cos\frac{\pi}{3} = \frac{(\hat{i}+\hat{k})\cdot(\hat{i}+\hat{j}+a\hat{k})}{\sqrt{2}\sqrt{1+1+a^2}}$

$$\Rightarrow \frac{1}{2} = \frac{1+a}{\sqrt{2}\sqrt{2+a^2}} \Rightarrow \frac{1}{4} = \frac{(1+a)^2}{2(2+a^2)}$$

$$\Rightarrow 2 + a^2 = 2(1 + a^2 + 2a) \Rightarrow a^2 + 4a = 0 \Rightarrow a = 0, -4$$

17. **(i)** **(b) :** Since, it rained only 5 days each year, therefore, probability that it rains on wedding day is $\frac{5}{365} = \frac{1}{73}$

(ii) **(c) :** The probability that it does not rain on wedding day $= 1 - \frac{1}{73} = \frac{72}{73} = \frac{360}{365}$

(iii) **(c) :** It is given that, when it actually rains, the weatherman correctly forecasts rain 90% of the time.

∴ Required probability $= \frac{90}{100} = \frac{9}{10}$

(iv) **(a) :** Let A_1 be the event that it rains on wedding day, A_2 be the event that it does not rain on wedding day and E be the event the weatherman predict rain.

Then we have, $P(A_1) = \frac{5}{365}$, $P(A_2) = \frac{360}{365}$,

$P(E/A_1) = \frac{9}{10}$ and $P(E/A_2) = \frac{1}{10}$

Required probability

$$= P(A_1/E) = \frac{P(A_1)\cdot P(E/A_1)}{P(A_1)\cdot P(E/A_1) + P(A_2)\cdot P(E/A_2)}$$

$$\frac{\frac{5}{365}\times\frac{9}{10}}{\frac{5}{365}\times\frac{9}{10}+\frac{360}{365}\times\frac{1}{10}} = \frac{45}{405} \approx 0.111$$

(v) **(a) :** Required probability $= 1 - P(A_1/E) = 1 - 0.111 = 0.889$

18. **(i)** **(c) :** Let $C(x)$ be the maintenance cost function, then $C(x) = 4000000 + 140x - 0.04x^2$

(ii) **(b) :** We have, $C(x) = 4000000 + 140x - 0.04x^2$
Now, $C'(x) = 140 - 0.08x$
For maxima/minima, put $C'(x) = 0$
$\Rightarrow 140 = 0.08x \Rightarrow x = 1750$

(iii) **(b) :** Clearly, from the problem statement we can see that we only want critical points that are in the interval $[0, 5000]$
Now, we have $C(0) = 4000000$
$C(1750) = 4122500$
and $C(5000) = 3700000$
∴ Maximum value of $C(x)$would be ₹ 4122500.

(iv) **(a) :** The complex must have 5000 apartments to minimise the maintenance cost.

(v) **(a) :** The minimum maintenance cost for each apartment would be ₹ 740.

19. Given, $f(x)$ is continuous at $x = 3$.

So, $\lim_{x\to 3} f(x) = f(3) \Rightarrow \lim_{x\to 3}\frac{(x+3)^2-36}{x-3} = k$

$$\Rightarrow \lim_{x\to 3}\frac{(x+3)^2-6^2}{x-3} = k$$

$$\Rightarrow \lim_{x\to 3}\frac{(x+3+6)(x+3-6)}{x-3} = k$$

$$\Rightarrow 3 + 3 + 6 = k \Rightarrow k = 12$$

20. Let $I = \int\frac{1}{\sqrt{1-\sin x}}dx = \int\frac{1}{\sqrt{1+\cos\left(\frac{\pi}{2}+x\right)}}dx$

$$= \int\frac{1}{\sqrt{2\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}}dx = \frac{1}{\sqrt{2}}\int\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$

$$= \frac{2}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4}+\frac{\pi}{8}+\frac{x}{4}\right)\right| + C$$

$$= \sqrt{2}\log\left|\tan\left(\frac{3\pi}{8}+\frac{x}{4}\right)\right| + C$$

OR

Let $I = \int \frac{dx}{1 - \sin 2x}$

Put $\tan x = t \Rightarrow dx = \frac{dt}{1+t^2}$ and $\sin 2x = \frac{2t}{1+t^2}$

$$\therefore \quad I = \int \frac{dt}{1+t^2-2t} = \int \frac{dt}{(t-1)^2} = \frac{-1}{t-1} + C$$

$$\Rightarrow \quad I = \frac{-1}{\tan x - 1} + C$$

21. We write, $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$

$$= \tan^{-1}\left(\frac{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right) + \sin^2\left(\frac{x}{2}\right) + 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\right)$$

$$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right] = \tan^{-1}\left[\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right] = \frac{\pi}{4} - \frac{x}{2}$$

OR

Let $\cos^{-1}\left(\frac{-3}{5}\right) = \theta$, where $\theta \in [0, \pi]$

Then, $\cos\theta = \frac{-3}{5}$

Since $\theta \in [0, \pi]$, we have $\sin\theta > 0$

$$\therefore \quad \sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\therefore \quad \sin\left[2\cos^{-1}\left(\frac{-3}{5}\right)\right] = \sin 2\theta = 2\sin\theta\cos\theta$$

$$= \left\{2 \times \frac{4}{5} \times \left(\frac{-3}{5}\right)\right\} = \frac{-24}{25}$$

22. Here, $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{(1,\,3)} = 4 - 18 + 26 - 10 = 2$$

Hence the equation of the tangent to (i) at (1, 3) is $y - 3 = 2(x - 1) \Rightarrow y = 2x + 1$

23. Given that, $\vec{a} = \hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 4\hat{k}$

$\therefore \quad \vec{a} + \vec{b} = 2\hat{i} + 8\hat{k}$ and $\vec{a} - \vec{b} = 2\hat{j}$

Let θ be the angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, then

$$\cos\theta = \frac{(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})}{|\vec{a} + \vec{b}||\vec{a} - \vec{b}|}$$

$$= \frac{(2\hat{i} + 0\hat{j} + 8\hat{k}) \cdot (0\hat{i} + 2\hat{j} + 0\hat{k})}{\sqrt{2^2 + 0^2 + 8^2}\sqrt{0^2 + 2^2 + 0^2}} = \frac{0 + 0 + 0}{\sqrt{4 + 64}\sqrt{4}} = 0$$

$$\Rightarrow \quad \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

24. The matrix will not be invertible if $\begin{vmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad 1(2 - 5) - a(1 - 10) + 2(1 - 4) = 0$

$\Rightarrow \quad -3 + 9a - 6 = 0 \Rightarrow a = 1$

25. The given line is $5x - 3 = 15y + 7 = 3 - 10z$

$$\Rightarrow \quad \frac{x - \frac{3}{5}}{\frac{1}{5}} = \frac{y + \frac{7}{15}}{\frac{1}{15}} = \frac{z - \frac{3}{10}}{-\frac{1}{10}}$$

Its direction ratios are $\frac{1}{5}, \frac{1}{15}, -\frac{1}{10}$

i.e., its direction ratios are proportional to 6, 2, –3.

Now, $\sqrt{6^2 + 2^2 + (-3)^2} = 7$

$\therefore \quad$ Its direction cosines are $\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}$.

OR

Any point on the line

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = r \text{ (say)} \qquad \text{...(i)}$$

is $(3r - 1, 5r - 3, 7r - 5)$.

Any point on the line

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = k \text{ (say)} \qquad \text{...(ii)}$$

is $(k + 2, 3k + 4, 5k + 6)$

For lines (i) and (ii) to intersect, we must have

$3r - 1 = k + 2, 5r - 3 = 3k + 4, 7r - 5 = 5k + 6$

On solving these, we get $r = \frac{1}{2}, k = -\frac{3}{2}$

$\therefore \quad$ Lines (i) and (ii) intersect and their point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

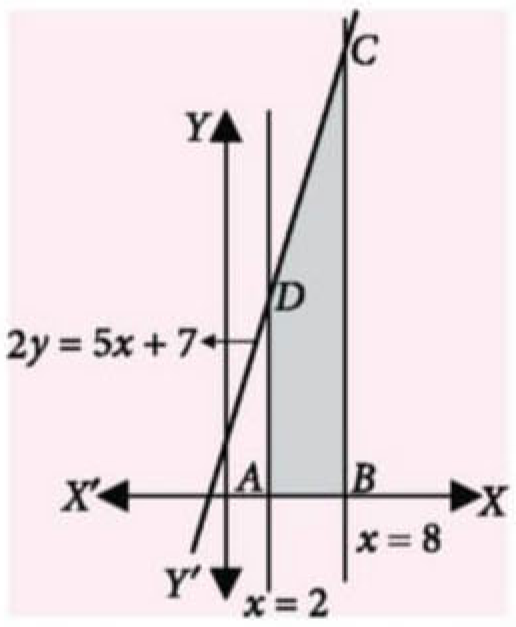
26. Let $u(x) = \sin^2 x$ and $v(x) = e^{\cos x}$.

Now, $\frac{du}{dx} = 2 \sin x \cos x$

and $\dfrac{dv}{dx} = e^{\cos x}(-\sin x) = -(\sin x)e^{\cos x}$

Thus, $\dfrac{du}{dv} = \dfrac{\frac{du}{dx}}{\frac{dv}{dx}} = \dfrac{2\sin x\cos x}{-\sin x\, e^{\cos x}} = -\dfrac{2\cos x}{e^{\cos x}}$

27. Let us draw the graph of given lines, as shown below:



$\therefore$ Required area (shown in shaded region)

$= \int_2^8 y\, dx = \int_2^8 \left(\dfrac{5x+7}{2}\right)dx$

$= \dfrac{1}{2}\left[\dfrac{5x^2}{2} + 7x\right]_2^8$

$= \dfrac{1}{2}\left[\left\{\dfrac{5(64)}{2} + 56\right\} - \left\{\dfrac{5(4)}{2} + 14\right\}\right]$

$= \dfrac{1}{2}[(160 + 56) - (10 + 14)] = \dfrac{1}{2}(216 - 24) = \dfrac{192}{2}$

= 96 sq. units.

28. Let E = the event of A speaking the truth and F = the event of B speaking the truth

Then, $P(E) = \dfrac{60}{100} = \dfrac{3}{5}$ and $P(F) = \dfrac{90}{100} = \dfrac{9}{10}$

Required probability = P (A and B contradicting each other)

$= P(E\bar{F}$ or $\bar{E}F) = P(E\bar{F}) + P(\bar{E}F)$

$= P(E) \cdot P(\bar{F}) + P(\bar{E}) \cdot P(F)$

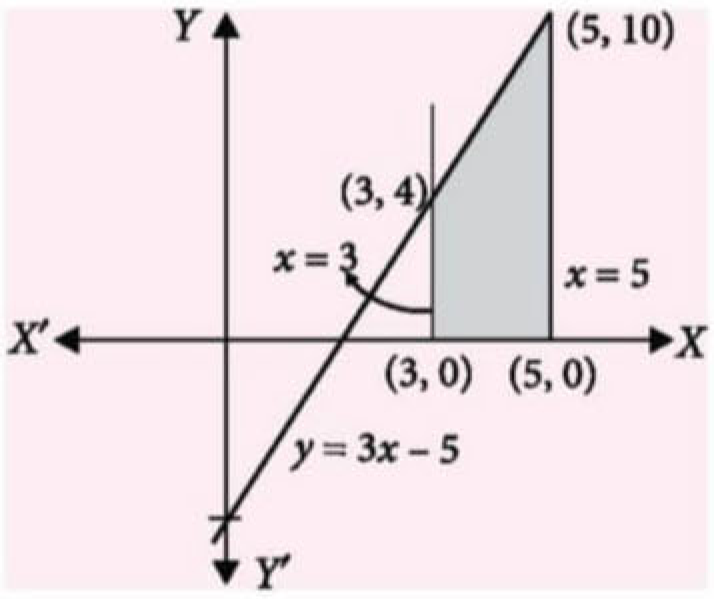
[$\because$ E and F are independent events]

$= P(E) \cdot [1 - P(F)] + [1 - P(E)] \cdot P(F)$

$= \dfrac{3}{5}\left(1 - \dfrac{9}{10}\right) + \left(1 - \dfrac{3}{5}\right) \cdot \dfrac{9}{10} = \dfrac{21}{50} = \dfrac{42}{100}$

Thus, A and B are likely to contradict each other in 42% cases.

29. We have, $y = 3x - 5$, $y = 0$, $x = 3$, $x = 5$



Required area $= \int_3^5 (3x - 5)\, dx = \left[\dfrac{3x^2}{2} - 5x\right]_3^5$

$= \dfrac{3}{2}(5^2 - 3^2) - 5(5 - 3)$

$= \dfrac{3}{2}(25 - 9) - 5 \times 2 = 24 - 10 = 14$ sq. units

30. Given $(a, b)\ R(c, d)$

$\Rightarrow\ ad = bc\ \forall\ (a, b), (c, d) \in N \times N$

Reflexive : Let $(a, b) \in N \times N$ when $a \in N, b \in N$

$(a, b)\ R\ (a, b) \Rightarrow ab = ba$, which is true

So, R is reflexive.

Symmetric : Let $(a, b), (c, d) \in N \times N$

$(a, b)\ R\ (c, d) \Rightarrow ad = bc$

$\Rightarrow\ cb = da \Rightarrow (c, d)\ R(a, b)$

So, R is symmetric.

Transitive : Let $(a, b), (c, d), (e, f) \in N \times N$

$(a, b)\ R(c, d) \Rightarrow ad = bc$

$(c, d)\ R\ (e, f)\ \Rightarrow cf = de$

$\Rightarrow\ adcf = bcde \Rightarrow af = be \Rightarrow (a, b)\ R\ (e, f)$

So, R is transitive.

Since R is reflexive, symmetric and transitive, so R is an equivalence relation.

31. Let $f(x) = a^x$ ...(i)

Now, $\lim\limits_{x\to 0} a^x = \lim\limits_{x\to 0}(a^x - 1 + 1) = \lim\limits_{x\to 0}\left(\dfrac{a^x - 1}{x} \times x + 1\right)$

$= \lim\limits_{x\to 0}\dfrac{a^x - 1}{x} \times \lim\limits_{x\to 0} x + \lim\limits_{x\to 0}(1) = \log a \times 0 + 1 = 1$

$\therefore\ \lim\limits_{x\to 0} a^x = 1$

Let c be an arbitrary real number.

Then $\lim\limits_{x\to c^-} f(x) = \lim\limits_{h\to 0} f(c - h)$

$= \lim\limits_{h\to 0} a^{c-h} = \lim\limits_{h\to 0} a^c \cdot a^{-h} = a^c \lim\limits_{h\to 0}\dfrac{1}{a^h}$

$= a^c \times 1 = a^c = f(c)$ [By (i)]

Likewise $\lim\limits_{x\to c^+} f(x) = a^c$

$\therefore\ \lim\limits_{x\to c^-} f(x) = \lim\limits_{x\to c^+} f(x) = f(c)$

$\therefore$ f is continuous at $x = c$, where c is an arbitrary real number.

$\therefore$ $f(x) = a^x$ is continuous at every point.

OR

Given, $f(0) = 2\log 3$ ...(i)

$\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0}\dfrac{(3^{\sin x} - 1)^2}{x\log(1 + x)}$

CLASS 12

LATEST PATTERN WITH
REDUCED SYLLABUS

2021 CBSE
BOARD EXAMS

ONE STOP SOLUTION
FOR ALL YOUR BOARD
EXAM WORRIES

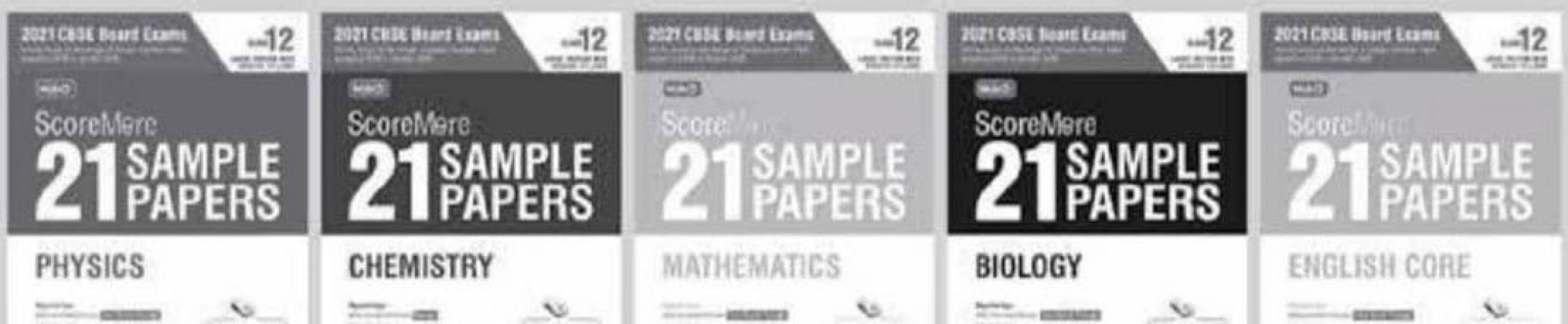
MTG
ScoreMore
21 SAMPLE PAPERS
CBSE BOARD EXAMS 2021

HIGHLIGHTS

- ✓ Objective Type
  MCQs | Assertion & Reason | Case Based Passage
- ✓ Subjective Type
  VSA | SA | LA
- ✓ 20 Sample Question Papers (SQPs) with BLUEPRINT as design issued by CBSE
- ✓ Latest CBSE Sample Question Paper 2020-21
- ✓ Self Evaluation Sheet included to check your readiness

ONLINE SUPPORT

- TIPS • EXAM ALERTS
- FORMULA BOOK
- CHAPTERWISE OBJECTIVE QUESTIONS
- 2020 CBSE SOLVED PAPER
- DETAILED SOLUTIONS OF UNSOLVED PAPERS


ScoreMore 21 SAMPLE PAPERS
PHYSICS
CHEMISTRY
MATHEMATICS
BIOLOGY
ENGLISH CORE

Available at all leading book shops throughout India.
For more information or for help in placing your order,
Call 0124-6601200 or e-mail:info@mtg.in

Strictly Based on the latest syllabus and design of
Sample Question Paper issued by CBSE in October 2020

$$= \lim_{x\to 0} \frac{\left(\frac{3^{\sin x}-1}{\sin x}\right)^2 \cdot \left(\frac{\sin x}{x}\right)^2}{(1/x)\log(1+x)}$$

$$= \lim_{x\to 0} \frac{\left(\frac{3^{\sin x}-1}{\sin x}\right)^2 \cdot \left(\frac{\sin x}{x}\right)^2}{\frac{\log(1+x)}{x}}$$

$$= \frac{(\log 3)^2 . (1)^2}{1} = (\log 3)^2 \qquad \text{...(ii)}$$

From (i) and (ii), $\lim_{x\to 0} f(x) \neq f(0)$

$\therefore$ f is discontinuous at $x = 0$.

32. Let side of square = x and radius of circle = r.
Perimeter of square = $4x$ and perimeter of circle = $2\pi r$
Let $4x + 2\pi r = k$

$$\Rightarrow \quad 4x = k - 2\pi r \Rightarrow x = \frac{k-2\pi r}{4} \qquad \text{...(i)}$$

Let A = Area of square + Area of circle

$$\Rightarrow \quad A = x^2 + \pi r^2 \Rightarrow A = \left(\frac{k-2\pi r}{4}\right)^2 + \pi r^2 \text{ [From (i)]}$$

Differentiating w.r.t. r, we get

$$\frac{dA}{dr} = 2\left(\frac{k-2\pi r}{4}\right)\left(-\frac{2\pi}{4}\right) + 2\pi r$$

$$\Rightarrow \quad \frac{dA}{dr} = \frac{-k\pi + 2\pi^2 r + 8\pi r}{4}$$

For maxima or minima, $\frac{dA}{dr} = 0$

$$\Rightarrow \quad -k\pi + 2\pi^2 r + 8\pi r = 0 \Rightarrow -k + 2\pi r + 8r = 0$$

$$\Rightarrow \quad k = 2\pi r + 8r \Rightarrow k = r(2\pi + 8) \Rightarrow r = \frac{k}{2\pi + 8}$$

Now, $\frac{d^2A}{dr^2} = \frac{2\pi^2 + 8\pi}{4} > 0$, so, local minima

From (i), $x = \frac{2\pi r + 8r - 2\pi r}{4} = 2r$

Hence, side of square is equal to diameter of circle when their combined area is least.

33. Let $I = \int_0^{3/2} |x \cos \pi x|\, dx$

$$\because \quad |x\cos \pi x| = \begin{cases} x\cos \pi x \ ; & 0 < x < \frac{1}{2} \\ -x\cos \pi x \ ; & \frac{1}{2} < x < \frac{3}{2} \end{cases}$$

$$\therefore \quad I = \int_0^{1/2} (x\cos \pi x)dx - \int_{1/2}^{3/2} x\cos \pi x\, dx$$

$$= \left[\frac{x\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right]_0^{1/2} - \left[\frac{x\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right]_{1/2}^{3/2}$$

$$= \left[\left(\frac{1}{2\pi}+0\right) - \left(0 + \frac{1}{\pi^2}\right)\right] - \left[\left(\frac{-3}{2\pi}+0\right) - \left(\frac{1}{2\pi}+0\right)\right]$$

$$= \frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{4}{2\pi} = \frac{5}{2\pi} - \frac{1}{\pi^2} = \frac{5\pi - 2}{2\pi^2}$$

34. We have, $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \Rightarrow \frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, with $P = \frac{1}{\sqrt{x}}$ and $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

$$\text{I.F.} = e^{\int P\,dx} = e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$$

$\therefore$ The solution is given by

$$ye^{2\sqrt{x}} = \int e^{2\sqrt{x}} \cdot \frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx + C$$

$$\Rightarrow \quad ye^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}}dx + C \Rightarrow ye^{2\sqrt{x}} = 2\sqrt{x} + C$$

$\Rightarrow \quad y = (2\sqrt{x} + C)e^{-2\sqrt{x}}$, which is the required solution.

OR

The given D.E. is $\frac{dy}{dx} + y\cot x = 2\cos x$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = \cot x$; $Q = 2\cos x$

Now, $\text{I.F.} = e^{\int P\,dx} = e^{\int \cot x\,dx} = e^{\log \sin x} = \sin x$

$\therefore$ The solution of equation (i) is given by

$$y\sin x = \int 2\cos x \cdot \sin x\, dx + c$$

$$= \int \sin 2x\, dx + c = -\frac{1}{2}\cos 2x + c$$

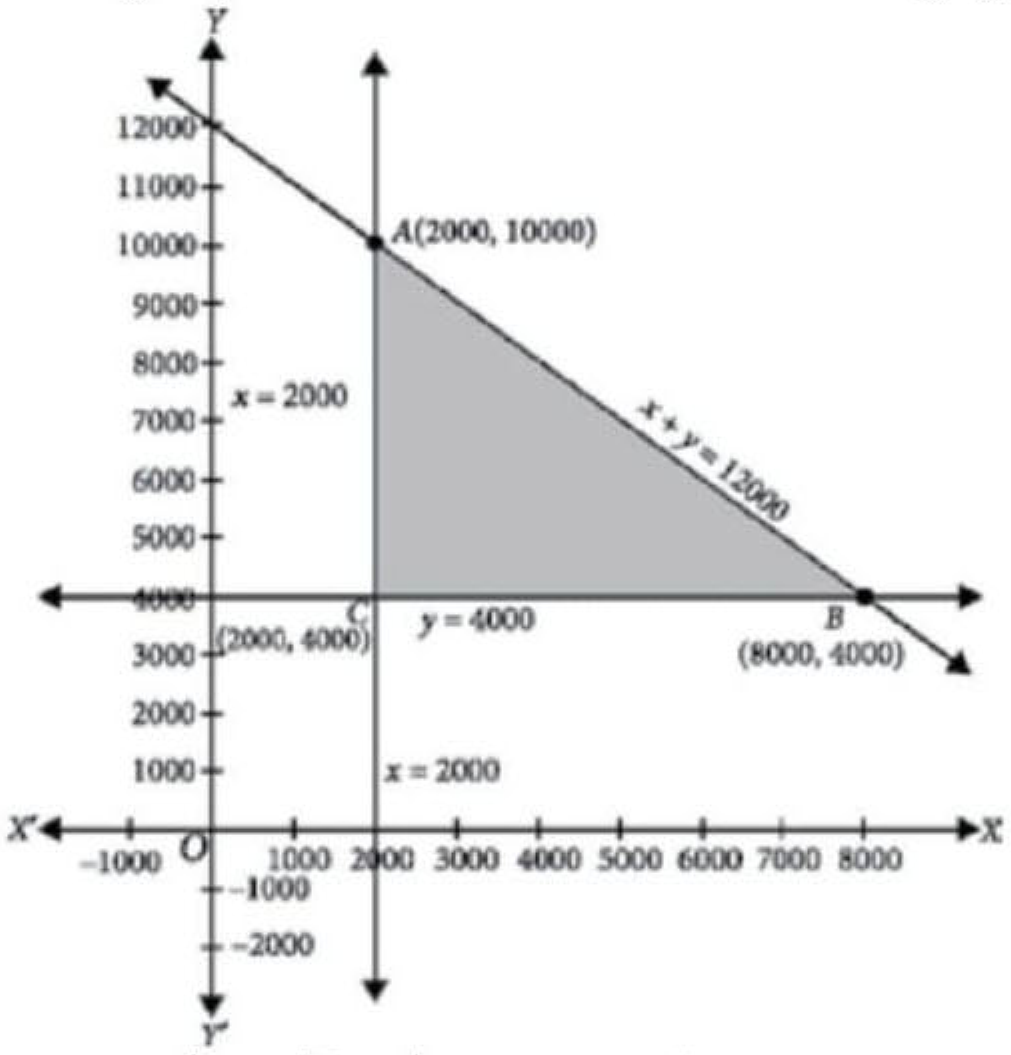
$\Rightarrow \quad y = -\frac{1}{2}\cos 2x \operatorname{cosec} x + c\operatorname{cosec} x$, which is the required solution of the given differential equation.

35. *Refer to Answer 25, page no. 97 of MTG CBSE Champion Mathematics, Class 12.*

36. Given L.P.P. is Maximize $Z = \frac{2x}{25} + \frac{y}{10}$ ... (i)
subject to constraints

$$x \geq 2000 \quad \text{... (ii)}$$
$$y \geq 4000 \quad \text{... (iii)}$$
$$\text{and } x + y \leq 12000 \quad \text{... (iv)}$$

On plotting inequalities (ii), (iii) and (iv), we get the shaded region, which is bounded in the following figure.



Now, we evaluate Z at the corner points $A(2000, 10000)$, $B(8000, 4000)$ and $C(2000, 4000)$.

Corner Points	**Value of** $Z = \frac{2x}{25} + \frac{y}{10}$
$A(2000, 10000)$	1160 (Maximum)
$B(8000, 4000)$	1040
$C(2000, 4000)$	560

Clearly, maximum value of Z is 1160 at $x = 2000$ and $y = 10000$.

OR

Refer to Answer 8, page no. 291-292 of MTG CBSE Champion Mathematics, Class 12.

37. We have, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$\therefore \quad A^2 = AA = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\cdot 3 + (-5)\cdot(-4) & 3\cdot(-5) + (-5)\cdot 2 \\ -4\cdot 3 + 2\cdot(-4) & -4\cdot(-5) + 2\cdot 2 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

Given $f(x) = x^2 - 5x - 14 \Rightarrow f(A) = A^2 - 5A - 14I_2$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 - 15 - 14 & -25 - (-25) - 0 \\ -20 - (-20) - 0 & 24 - 10 - 14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$$\Rightarrow \quad A^2 = 5A + 14I_2 \quad \text{...(i)}$$

$$\therefore \quad A^3 = AA^2 = A(5A + 14I_2) \quad \text{(Using (i))}$$

$$= A(5A) + A(14I_2) = 5AA + 14(AI_2)$$

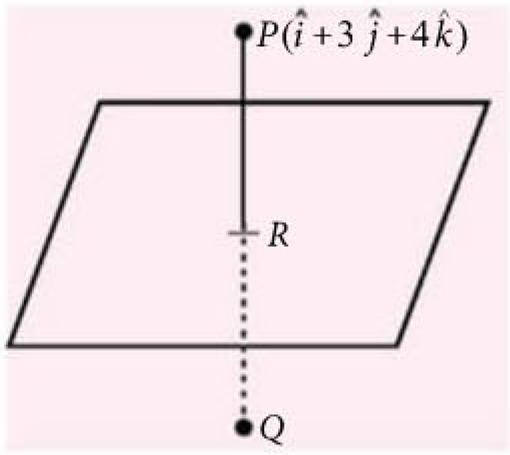
$$= 5A^2 + 14A = 5\,(5A + 14I_2) + 14A \text{ (Using (i))}$$

$$= 39A + 70I_2 = 39\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} + 70\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 117 & -195 \\ -156 & 78 \end{bmatrix} + \begin{bmatrix} 70 & 0 \\ 0 & 70 \end{bmatrix} = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

OR

Refer to Answer 98, page no. 70-71 of MTG CBSE Champion Mathematics, Class 12.



38. Let Q be the image of the point $P(\hat{i} + 3\hat{j} + 4\hat{k})$ in the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$

Then, PQ is normal to the plane. Since PQ passes through P and is normal to the given plane, therefore equation of line PQ is

$\vec{r} = (\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

Since Q lies on line PQ, so let the position vector of Q be

$(\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$= (1 + 2\lambda)\hat{i} + (3 - \lambda)\hat{j} + (4 + \lambda)\hat{k}$

Since, R is the mid-point of PQ. Therefore, position vector of R is

$$\frac{[(1 + 2\lambda)\hat{i} + (3 - \lambda)\hat{j} + (4 + \lambda)\hat{k}] + [\hat{i} + 3\hat{j} + 4\hat{k}]}{2}$$

$$= (\lambda + 1)\hat{i} + \left(3 - \frac{\lambda}{2}\right)\hat{j} + \left(4 + \frac{\lambda}{2}\right)\hat{k}$$

Since R lies on the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$

$$\Rightarrow \left\{(\lambda+1)\hat{i} + \left(3 - \frac{\lambda}{2}\right)\hat{j} + \left(4 + \frac{\lambda}{2}\right)\hat{k}\right\}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$$

$$\Rightarrow \quad 2\lambda + 2 - 3 + \frac{\lambda}{2} + 4 + \frac{\lambda}{2} + 3 = 0 \;\Rightarrow\; \lambda = -2$$

Thus, the position vector of Q is

$(\hat{i} + 3\hat{j} + 4\hat{k}) - 2(2\hat{i} - \hat{j} + \hat{k}) = -3\hat{i} + 5\hat{j} + 2\hat{k}$.

OR

Refer to Answer 112, page no. 278-279 of MTG CBSE Champion Mathematics, Class 12.

◆

mtG

Now, savings of up to ₹920* with MTG's magazine subscription plans!

*On cover price of ₹40/-

Over 1.2 Cr readers. Since 1982.

Our new offers are here!
Pick the combo best suited for your needs. Fill-in the Subscription Form at the bottom and mail it to us today. If in a rush, log on to *www.mtg.in* now to subscribe online.

For JEE (Main & Advanced), NEET and BOARDS

About MTG's Magazines

Perfect for students who like to prepare at a steady pace, MTG's magazines (**Physics For You, Chemistry Today, Mathematics Today & Biology Today**) ensure you practice bit by bit, month by month, to build all-round command over key subjects. Did you know these magazines are the only source for solved test papers of all national and state level engineering and medical college entrance exams?

- Practice steadily, paced month by month, with very-similar & model test papers
- Self-assessment tests for you to evaluate your readiness and confidence for the big exams
- Content put together by a team comprising experts and members from MTG's well-experienced Editorial Board
- Stay up-to-date with important information such as examination dates, trends & changes in syllabi
- All-round skill enhancement – confidence-building exercises, new studying techniques, time management, even advice from past JEE/NEET/ AIIMS toppers
- Bonus: Exposure to competition at a global level, with questions from International Olympiads & Contests

Lifetime Subscription Plan for teachers, and special schemes and offers available for libraries and coaching institutes. SMS MTG to 8800255334 to learn more.

SUBSCRIPTION FORM

Confirm your choice by placing ☑tick-marks in relevant boxes.

		27 months	15 months	9 months
Plan 1: **Individual magazines P, C, M, B**	☐ Physics ☐ Chemistry ☐ Mathematics ☐ Biology	☐ ₹850 *(save ₹ 230)*	☐ ₹500 *(save ₹ 100)*	☐ ₹300 *(save ₹ 60)*
Plan 2: **Combo of 3**	☐ PCM ☐ PCB	☐ ₹2500 *(save ₹ 740)*	☐ ₹1400 *(save ₹ 400)*	☐ ₹900 *(save ₹ 180)*
Plan 3: **PCMB Combo**		☐ ₹3400 *(save ₹ 920)*	☐ ₹1900 *(save ₹ 500)*	☐ ₹1200 *(save ₹ 240)*
Courier Charges *Add to your subscription amount for quicker & reliable delivery*		☐ ₹600	☐ ₹450	☐ ₹240

Recommended by *(Optional)*

____
Name of your teacher

Teacher's Mobile # ☐☐☐☐☐☐☐☐☐☐

Note: Magazines are despatched by Book-Post on 4^{th} of every month (each magazine separately).

Name: _______________

Complete Postal Address: _______________

Pin Code ☐☐☐☐☐☐

Mobile # ☐☐☐☐☐☐☐☐☐☐

Other Phone # 0 ☐☐☐☐☐☐☐☐☐☐☐

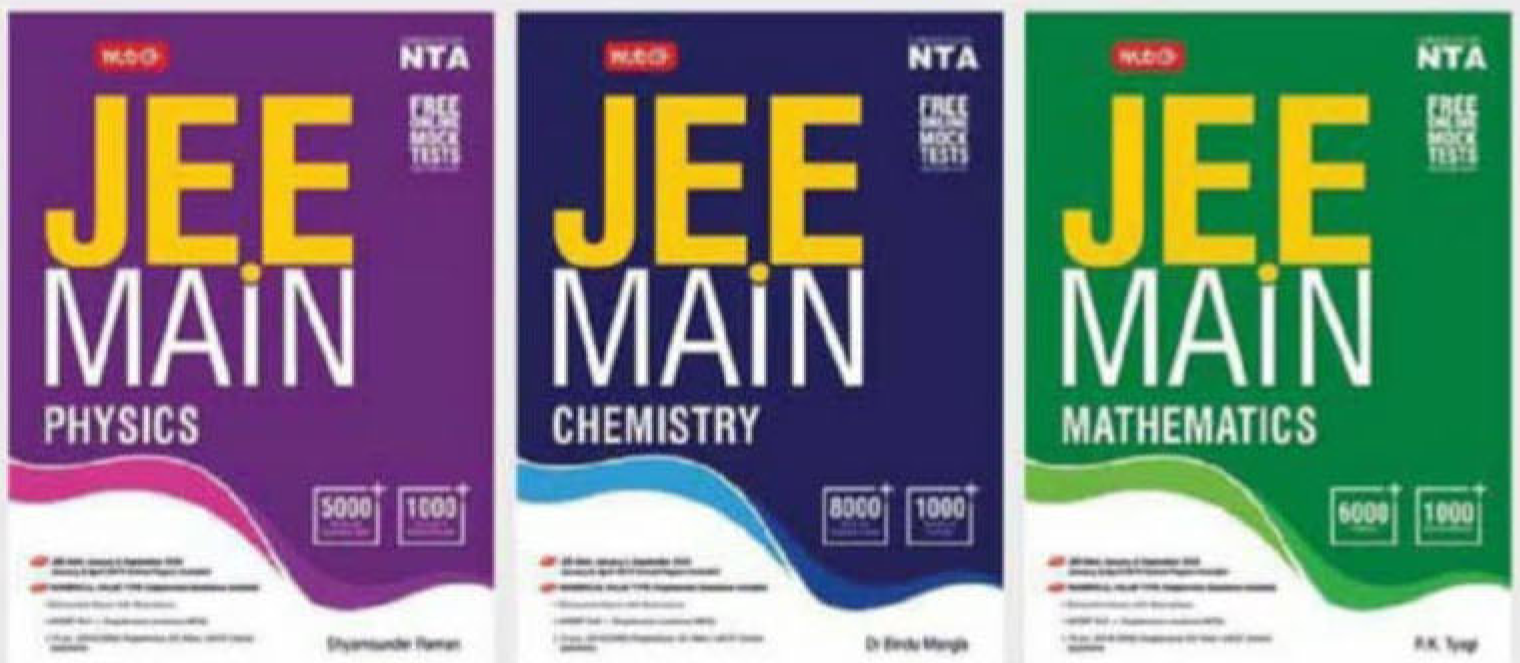
Email _______________

Enclose Demand Draft favouring
MTG Learning Media (P) Ltd, payable at New Delhi.
Mail this Subscription Form to Subscription Dept.,
MTG Learning Media (P) Ltd, Plot 99, Sector 44, Gurugram -122 003 (HR).

E-mail *subscription@mtg.in*. Visit *www.mtg.in* to subscribe online. **Call (0)1800-10-38673 for more info.**
Get digital editions of MTG Magazines on *http://digital.mtg.in/*

mtG
Success Made Simple

CRACKING
JEE
IS THIS SIMPLE


JEE MAIN PHYSICS
JEE MAIN CHEMISTRY
JEE MAIN MATHEMATICS
NTA
FREE ONLINE MOCK TESTS

ALL THE HELP YOU NEED TO CRACK JEE:

- Based on latest pattern of JEE Main • Covers the entire syllabus
- Full of graphic illustration for deep understanding of concepts
- Level wise MCQs with detailed solutions
- NCERT Drill MCQs framed from NCERT Books
- Previous 10 Years' MCQs (2018-2009) of JEE Main/AIEEE
- Numerical Value Type chapterwise questions included
- JEE Main January & September 2020 and January & April 2019 Solved Papers Included

Available at all leading book shops throughout India. To buy online visit www.mtg.in

For more information or for help in placing your order, call 0124-6601200 or email info@mtg.in

Registered R.N.I. No. 40700/1983 Published on 1st of every month Postal Regd. No. DL-SW-01/4045/21-23
Lic. No. U(SW)-29/2021-23 to post without prepayment of postage N.D.P.S.O.N.D-1 on 2-3rd same month

Aakash
Medical | IIT-JEE | Foundations

www.aakash.ac.in

JOIN US IF YOU

DREAM BIG.
VALUE DISCIPLINE.
BELIEVE IN YOURSELF.
ARE FOCUSED ON YOUR GOAL.

Because **Aakash** is a platform that will transform you into an extra-ordinary achiever by partnering and working hard with you, thereby nurturing you to achieve your career goal.

Call us to know more about how we can help you fulfill your dream of becoming an **IITian** or a **Doctor!**

Embrace the new way of learning with Aakash Hybrid
Best of Classroom & Online Learning

Courses available for Class VII to XII studying & XII passed students

SCAN FOR NEAREST CENTRE & CONTACT DETAILS

TOLL-FREE: 1800-102-2727
Give a missed call: 9599280605

Registered Office: Aakash Tower, 8, Pusa Road, New Delhi-110005. **Ph.:** (011) 47623456 | **E-mail:** care@aesl.in | **SUNDAY OPEN**

www.aakash.ac.in | instagram.com/aakasheducation | facebook.com/aakasheducation | twitter.com/aakash_twitted | youtube.com/AakashEducation | aakash.ac.in/blogs